Optimal Design of a Permanent Magnet Synchronous Motor to Improve Torque and Demagnetization Characteristics

Yong-Min You\textsuperscript{1,*} and Dae-Won Chung\textsuperscript{2}

Department of \textsuperscript{1}Automotive Engineering and \textsuperscript{2}Electrical Engineering, Honam University, Gwangju 62399, Korea

(Received 30 May 2017, Received in final form 14 July 2017, Accepted 19 July 2017)

This study proposes an unequal air-gap model for PMSMs with the objective of reducing the torque ripple and improving the demagnetization characteristics. Shape optimization is performed to optimize the design variables of the unequal air-gap model, which are the shape of the unequal air-gap and the angle between the V-shaped permanent magnets. An optimization process was performed by the Kriging model based on Latin Hypercube Sampling (LHS), and Genetic Algorithm (GA). Finite element analysis (FEA) was also utilized to analyze the torque and demagnetization characteristics. As optimal design results, the torque ripple of the optimal model was reduced by 7.7\% and the demagnetization temperature was improved by 2.2\%.

Keywords : permanent magnet synchronous motor, unequal air-gap, torque ripple, demagnetization, finite element analysis, optimal design

1. Introduction

The study of the torque ripple of Permanent Magnet Synchronous Motors (PMSMs) has been rapidly progressing, which has resulted in the improvement in the noise and vibrations of electric vehicles. There are several ways to reduce the cogging torque such as increasing the air-gap width and pole/slot number, notches in rotor and stator teeth, and skewing of rotor and stator [1-3]. The skewing of rotor and stator has been applied to the traction motor of GM Spark and Ford Focus. Recently, new methods have been developed to improve the cogging torque, which are different widths of the permanent magnet, unequal teeth width, and skewing of the permanent magnets [4-6]. However, all the above mentioned methods have disadvantages in terms of material costs and productivity.

PMSM has been widely used in electric vehicles due to its high power density. The demand for Dysprosium-free (Dy-free) permanent magnet for PMSMs has been increasing because of a limited supply of rare earth metals. However, the demagnetization temperature is a critical factor in the use of PMSMs for electric vehicles. The demagnetization characteristics of PMSMs have become more significant because Dy-free permanent magnet has lower coercivity [7].

This study proposes an unequal air-gap model for PMSMs with the objective of reducing the torque ripple and improving the demagnetization characteristics, which are significant parameters in the application of PMSMs in electric vehicles. Shape optimization is performed to optimize the design variables of the unequal air-gap model. The optimal design variables are the shape of the unequal air-gap and the angle between the V-shaped permanent magnets. An optimization process was performed by Kriging model based on Latin Hypercube Sampling (LHS) and Genetic Algorithm (GA). Finite element analysis (FEA) was also utilized to analyze the torque and demagnetization characteristics.
permanent magnets is applied to reduce the torque ripple. Water cooling system is also applied considering high current.

Although a PMSM with non-skewing permanent magnets is considered to reduce material costs and to improve productivity, the torque ripple is 10 %, which is impossible to mass-produce. Dy-free permanent magnets are also reviewed to reduce material costs, but the demagnetization temperature is 146°C, which does not satisfy the requirement of PMSMs for electric vehicles. The required minimum demagnetization temperature of PMSM for electric vehicles is 150°C.

To apply non-skewing and Dy-free permanent magnets, an unequal air-gap is applied, as shown in Fig. 2(b), which can reduce the torque ripple and improve the demagnetization temperature. An optimal design is also performed for further improvement.

### 2.2. Optimal design process

Figure 3 shows an optimal design process to optimize effectively the PMSM. The objective function and constraints are established, and then decide the optimal design variables and its boundary. LHS is applied as a sampling method of the design of experiment (DOE), and FEA is utilized to analyze the fixed sampling points by

![Fig. 3. Optimal design process.](image)

---

**Table 1. Specifications of analysis model.**

<table>
<thead>
<tr>
<th>Items</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. output power</td>
<td>kW</td>
<td>45</td>
</tr>
<tr>
<td>Max. torque</td>
<td>N·m</td>
<td>120</td>
</tr>
<tr>
<td>Rated speed</td>
<td>rpm</td>
<td>4,000</td>
</tr>
<tr>
<td>Max. speed</td>
<td>rpm</td>
<td>12,000</td>
</tr>
<tr>
<td>DC link voltage</td>
<td>Vdc</td>
<td>343</td>
</tr>
<tr>
<td>Peak current</td>
<td>Arms</td>
<td>150</td>
</tr>
<tr>
<td>Continuous current</td>
<td>Arms</td>
<td>58</td>
</tr>
<tr>
<td>No. of poles</td>
<td>Poles</td>
<td>8</td>
</tr>
<tr>
<td>No. of slots</td>
<td>EA</td>
<td>48</td>
</tr>
<tr>
<td>Outer diameter</td>
<td>mm</td>
<td>185</td>
</tr>
<tr>
<td>Lamination</td>
<td>mm</td>
<td>72</td>
</tr>
<tr>
<td>Air gap</td>
<td>mm</td>
<td>0.7</td>
</tr>
<tr>
<td>Winding spec.</td>
<td>-</td>
<td>ø0.85, 48 turns/phase</td>
</tr>
<tr>
<td>Core material</td>
<td>-</td>
<td>Posco 27PNF1400</td>
</tr>
<tr>
<td>P.M. material</td>
<td>-</td>
<td>Shinetsu N50SH-GF</td>
</tr>
<tr>
<td>Residual flux density</td>
<td>-</td>
<td>13,800~14,400</td>
</tr>
<tr>
<td>Coercivity</td>
<td>Oe</td>
<td>1,671~21,000</td>
</tr>
</tbody>
</table>

---

![Fig. 1. (Color online) PMSM for electric vehicle.](image)

![Fig. 2. Shape of rotor and stator.](image)
LHS. The Kriging model is used to approximate the objective and constraints functions using DOE results. We can finally get the optimal design values by approximated Kriging model and GA, which is an optimization algorithm.

2.3. Optimal design techniques

The Kriging model is a group of geostatistical techniques used to interpolate the value of a random field. In this use of the Kriging model, the estimated equation was defined to eliminate bias and thereby minimize error variance [8]. The Kriging model is a weighted linear combination as follows.

\[ z^* = \sum_{i=1}^{n} \lambda_i z_i \]  

where \( z^* \) is an estimated point using the Kriging model, \( n \) is the total number of experiments, \( \lambda_i \) is the weight value function, and \( z_i \) is the experimented point.

The minimized error deviation of the Kriging model can be expressed as

\[ \text{Minimize} \quad \sigma_{OK}^2 = \sigma^2 - 2 \sum_{i=1}^{n} \lambda_i \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij}^2 \]  

With a constraint \( 1 - \sum_{i=1}^{n} \lambda_i = 0 \)

where \( \sigma_{OK}^2 \) is an error variance of the Kriging model, \( \sigma^2 \) is a variance of \( z_0 \), \( \sigma_i^2 \) is a covariance of \( z_0 \) and \( z_i \), \( \sigma_{ij}^2 \) is a covariance of \( z_i \) and \( z_j \), \( z_0 \) is a real value to predict.

When the error deviation of the Kriging model is minimized, it can be expressed as

\[ L(\lambda_1, \lambda_2, \ldots, \lambda_n; \omega) = \sigma^2 - 2 \sum_{i=1}^{n} \lambda_i \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j \sigma_{ij}^2 + 2\omega \left( 1 - \sum_{i=1}^{n} \lambda_i \right) \]  

where \( L(\lambda_1, \lambda_2, \ldots, \lambda_n; \omega) \) is a Lagrange objective function, \( \omega \) is a Lagrange factor and the coefficient 2 is used for convenience.

The objective function is calculated by a partial derivative of Lagrange factor with respect to \( \lambda \) and \( \omega \) as follows.

\[ \sum_{i=1}^{n} \lambda_i \sigma_i^2 - \omega = \sigma_i^2, l = 1, 2, \ldots, n \]  

The error deviation of the Kriging model can be expressed as

\[ \sigma_{OK}^2 = \text{Var}(z) - \sum_{i=1}^{n} \lambda_i \text{Cov}(z_i, z_i) + \omega = \sigma^2 - \sum_{i=1}^{n} \lambda_i \sigma_i^2 + \omega \]  

LHS is a space filling design, which tends to uniformly sample points in the whole parameter space. To effectively construct the Kriging model, LHS is also used for improved accuracy over random sampling and stratified sampling to estimate the means, deviations and distribution functions of an output. Moreover, it ensures that each of the input variables represent all portions of its range.

The difference between random sampling and LHS technique in the two-dimensional plane is shown in Fig. 4. A new sample point of random sampling is created without taking into account the previously generated sample points. Therefore, random sampling does not require upfront information on the necessary number of sampling points. However, the LHS technique determines the total number of sampling points before placing a sample point with matrix information.

As the final step of the optimization, GA was applied to find the optimal points of the design variables. GA is a global optimization algorithm, which is based on the evolution of the natural world. GA describes the possible solution as a specific form of the data structure. A better solution can be found by gradually transforming the possible solution. The GA will repeat the major computations, such as evaluation, creation of a mating pool, crossover operation, and mutation until the objective function meets the desired level or the maximum number of repetitions is reached.

2.4. Optimal design conditions

To improve the torque ripple and demagnetization temperature, the objective function and constraints are established as Equation (6) and (7), respectively. The objective function is to minimize the torque ripple of PMSM. The constraints are the demagnetization temperature of permanent magnets and the maximum torque of PMSM. To satisfy the objective function and the constraints, the design variables are decided as shown in Fig. 5. The design variables are the angle (X1) between the V-shaped permanent magnets and the dimensions (X2, X3) concerned with the shape of an unequal air-gap. The total number of the DOE is 15, taking into account the number of the design variables.
Optimal Design of a Permanent Magnet Synchronous Motor to Improve Torque

Yong-Min You and Dae-Won Chung

• Objective function

Minimize the torque ripple

(6)

• Constraints

Demagnetization temperature of the permanent magnets

\[ \geq 150 \, ^\circ \text{C} \]

Max. torque \( \geq 126 \, \text{N\cdotm} \) (5 % margin of target max. torque, which is 120 N\cdotm)

(7)

2.5. Optimal design result

The optimal design was performed using PIANO, which is a commercial optimization program. The optimal design variables were well converged after 200 iterations. Optimal design results are shown in Table 2. The angle (X1) between the V-shaped permanent magnets was increased by 7.6° (from 116° to 123.6°), and the dimensions (X2, X3) of the unequal air-gap were also optimized.

2.5.1. Back EMF characteristics

It is easier to control a waveform that has been improved to a sinusoidal wave. The magnetic flux can flow smoothly within the magnetic circuit through the application of an unequal air-gap. As a result, the back EMF waveform was improved to a sinusoidal wave as shown in Fig. 6. The total harmonic distortion (THD) of the back EMF of the optimal model was 5.5 %, which was an improvement of approximately 2 % compared to 7.56 % of the non-skew basic model.

2.5.2. Torque characteristics

Figure 7 shows the torque characteristics when the

Table 2. Comparison of design variables.

<table>
<thead>
<tr>
<th>Items</th>
<th>Unit</th>
<th>Non-skew basic model</th>
<th>Unequal air-gap basic model</th>
<th>Unequal air-gap optimal model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design</td>
<td></td>
<td>116</td>
<td>122</td>
<td>123.6</td>
</tr>
<tr>
<td>variables</td>
<td></td>
<td></td>
<td>1.04</td>
<td>1.99</td>
</tr>
<tr>
<td>X1</td>
<td>°</td>
<td></td>
<td>0.7</td>
<td>1.91</td>
</tr>
<tr>
<td>X2</td>
<td>mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X3</td>
<td>mm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5. (Color online) Optimal design variables.

Fig. 6. (Color online) Back EMF characteristics.

Fig. 7. (Color online) Torque characteristics.
maximum current is applied at 900 rpm and the control angle is 47°. As shown in Fig. 7(a), the torque ripple of the non-skew basic model was approximately 10 %, which was difficult to replicate in mass production. Reduction of the torque ripple is possible by optimizing the magnetic flux distribution and reducing the magnetic saturation and space harmonics. The torque ripple was reduced to 2.9 % by applying the unequal air-gap and it was reduced to 2.3 % by the optimal design. There is a trade-off between the torque ripple and the maximum torque. Although the maximum torque was reduced slightly, it satisfied the constraint that it should be more than 126 N·m.

Figure 8 shows the harmonic analysis result of the torque. The ordinal number of harmonics influencing the torque ripple is a number that is a multiple of six. The harmonic analysis result of the three models showed that the values of the 12th harmonic were the greatest. The 12th harmonic of the non-skew basic model, unequal air-gap basic model, and unequal air-gap optimal model were 48 %, 36 %, and 17 %, respectively. The 12th harmonic element of the optimal model was distributed over the 6th and 24th harmonics in the optimal design.

2.5.3. Demagnetization characteristics

The management standard for the demagnetization property of the PMSM for electric vehicles is very strict. GM manages the B-H characteristics of permanent magnets at 140°C. Hyundai Motor Co. and LG Electronics Inc. demand that permanent magnets must not be demagnetized at 150°C. A demagnetization analysis was performed under the maximum current and control angle of 150 A and 80°, respectively.

The analysis results were represented by the flux density distribution of the permanent magnets. However, the values of the flux density of permanent magnets can be converted to temperature values using the demagnetization curves of the permanent magnet. Figure 9 shows the demagnetization curves of the N50SH-GF grade of Shinetsu. The Knee points under the conditions of 140°C and 170°C are 0.4 T and 0.6 T, respectively. The simulated values of the flux density of permanent magnets can be converted to temperature values using the values of the Knee points, as established in Equation (8).

\[
\text{Temperature of P. M.} = 140°C \times \frac{\text{Flux density of P. M. T} - 0.4 T}{0.6 T - 0.4 T} + (170°C - 140°C)
\]

with a constraint: \(0.4 \text{T} \leq \text{Flux density of P. M.} \leq 0.6 \text{T}\)
Figure 10 shows the demagnetization analysis results of the three models. The inner edges of the permanent magnets were vulnerable to demagnetization because the magnetic flux density at these places was the lowest. The

![Figure 10. (Color online) Demagnetization characteristics.](image)

Figure 11. (Color online) Speed-Torque characteristics.

![Figure 11. (Color online) Speed-Torque characteristics.](image)

Figure 12. (Color online) Efficiency map.

![Figure 12. (Color online) Efficiency map.](image)
demagnetization temperature of the non-skew basic model was 146.8°C because the minimum flux density is 0.4452 T. The demagnetization characteristic of the unequal air-gap optimal model was improved to 150°C by optimizing the angle between the V-shaped permanent magnets.

2.5.4. S-T characteristics and efficiency map

Figure 11 shows the speed-torque characteristics of the three models. The maximum torque is 120 N·m in the constant torque region until 4,000 rpm, which is the rated speed. The maximum power is 45 kW in the constant power region. The analysis results showed that the performance of the three models have similar characteristics. The efficiency map characteristics of the three models are shown in Fig. 12. The maximum efficiency of the three models were approximately 97.2~97.3 % around the rated speed. The efficiency distributions in the entire operation region also showed no significant differences.

2.5.5. Optimal design result

Table 3 shows the optimal design result of the three models. The THD of the back EMF of the optimal model was improved 2.1 % compared to the non-skew basic model. The torque ripple of the optimal model was reduced by 7.7 % and the demagnetization temperature was improved by 2.2 % compared to the non-skew basic model. Although the maximum torque of the optimal model was reduced slightly, it satisfied the constraint of 126 N·m. The maximum efficiency of the optimal model was almost equal to the initial model.

<table>
<thead>
<tr>
<th>Items</th>
<th>Unit</th>
<th>Non-skew basic model</th>
<th>Unequal air-gap basic model</th>
<th>Unequal air-gap optimal model</th>
</tr>
</thead>
<tbody>
<tr>
<td>THD of back EMF</td>
<td>%</td>
<td>7.6</td>
<td>5.4</td>
<td>5.5</td>
</tr>
<tr>
<td>Torque ripple</td>
<td>%</td>
<td>10.0</td>
<td>2.9</td>
<td>2.3</td>
</tr>
<tr>
<td>Max. torque</td>
<td>N·m</td>
<td>137.5</td>
<td>133.5</td>
<td>132.5</td>
</tr>
<tr>
<td>Demagnetization temperature</td>
<td>°C</td>
<td>146.8</td>
<td>148.6</td>
<td>150.0</td>
</tr>
<tr>
<td>Max. efficiency</td>
<td>%</td>
<td>97.2</td>
<td>97.3</td>
<td>97.3</td>
</tr>
</tbody>
</table>

The optimization results showed that the torque ripple was reduced because the flux distribution was optimized, and the magnetic saturation and space harmonic was diminished. The demagnetization temperature was improved by optimizing the angle between the V-shaped permanent magnets. The torque ripple of the optimal model was reduced by 7.7 % and the demagnetization temperature was improved by 2.2 %.

The proposed method is expected to reduce the material costs and increase the productivity because there is no need to apply skewing of the permanent magnets. Moreover, further material cost reduction is possible by applying Dy-free permanent magnets. In conclusion, this study suggests that applying an unequal air-gap and optimized design process to the PMSM for an electric vehicle can improve the performance and contribute to cost reduction as well as productivity.

3. Conclusion

This paper presented an unequal air-gap structure and optimal design of the PMSM for electric vehicles that can reduce the torque ripple and improved the demagnetization temperature. The angle between the V-shaped permanent magnets and the optimal dimensions of the unequal air-gap were determined using the Kriging model based on LHS and GA.

Acknowledgement

This study was supported by research fund from Honam University, 2015.

References