ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
Journal of Magnetics 23(1), 11-17 (2018) https://doi.org/10.4283/JMAG.2018.23.1.011

Validation of Wireless Power Transfer by using 3D Representation of Magnetically Coupled Resonators Considering Peak Efficiency

Nur Bekiroglu*, Ali Agcal, and Selin Ozcira

Electrical Engineering Department, Yildiz Technical University, Davutpasa, 34220, Istanbul, Turkey

(Received 22 June 2017, Received in final form 13 March 2018, Accepted 14 March 2018)

This paper focuses on wireless power transfer system based on magnetic resonance coupling which involves creating a resonance and transferring the power without radiating electromagnetic wave outwith the critical distance. Modelling with Ansys® Maxwell 3D software provides the means to observe the main field quantities with its post-processing capability. Therefore mathematical expressions of optimal coupling coefficients are analyzed by considering mutual coupling model which is presented along with a derivation of key system identifiers such as transmission distance, characteristic impedance and resonance frequency. The effectiveness of the system is analyzed by exciting the resonators with sinusoidal voltage source. Ansys® Maxwell 3D software is utilized to solve equivalent circuit and also to calculate mutual inductance and characteristic impedance according to air gap variations. Resonance frequency is a key parameter in system design whose value can be changed according to distance between resonators. The peak efficiency is analyzed depending on different air gap values for various characteristic impedances at optimum resonance frequency. In this study, modelling resonators in 3D has been constituted correspondingly. The approach demonstrated in this paper allows fixed load receiver to be moved to different orientation within the range of critical coupling distance and approximately efficiency of 70 %.

Keywords : coupling coefficient, peak efficiency, magnetically coupled resonators, wireless power transfer

1. Introduction

Wireless power transfer, having become one of the most widely researched topics in recent years, is now one of the fields where many researchers express various points of view among which WPT via coupled magnetic resonance has many merits [1-4]. Witricity, which is a wireless power transfer technology based on the concept of near field and strongly coupled magnetic resonance, was reported by Massachusetts Institute of Technology (MIT) research team [5]. The fundamental principle of this technology relies on the fact that same frequency resonant objects exchange energy with each other whilst non-resonant objects interact with each other via magnetic fields [6]. In order to have power transferred from primary coil to secondary one effectively, two conditions have to be fulfilled. The first condition is to have a very compact coil size while utilizing high frequency for better energy transmission. The second condition, on the other hand, is to have good magnetic coupling between two coils. Instead of spreading radiative electromagnetic field, the resonator fills enclosure with non-radiative magnetic field while oscillating at MHz range frequencies so that efficient power exchange between source and device resonators could be accomplished. These two resonators are defined to be critically coupled once their resonance frequencies are combined together to form one characteristic at a specific distance which is also called as critically coupled distance [7]. The result of this kind of operation is the ability to transfer power through medium distance with small mutual inductance.

It is commonly accepted to use a series resonating circuit as an equivalent circuit of a resonator to conduct analysis for resonant frequency [8]. There have been various attempts to find a sufficiently precise model [9, 10]. Within those researches, scattering parameters have been calculated by two port network theory utilizing network analyzer [11]. There are so many efforts involved such as modification circuits and magnetic design of core properties [12, 13]. Modeling of wireless power transfer

---

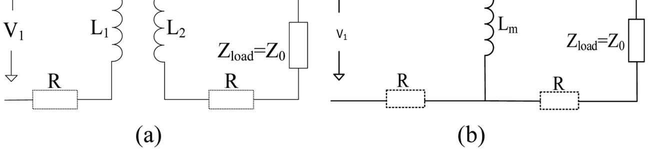
*Corresponding author: Tel: +905323237657
Fax: +902123835858, e-mail: nbekir67@gmail.com

system with copper resonators by utilizing electrical circuit theory, it will be essential to calculate the parameters of the resonators.

The effect of magnetic resonance circuit on the wireless power transfer system is not only affected by the parameters of the system itself but also by the distance between these two coils. To maximize efficiency of the transmission, the effects of operating parameters on the transmission performance, such as optimized load value, critical coupling and quality factor, are examined in the terms of figure of merit. Therefore in this study, depending on the air gap values as well as the selected frequency, the behavior of the system under various load conditions has been analyzed and the system efficiency plotted along figure-of-merit by using equivalent circuit model and mutual inductance theory given in [5]. The parameters of equivalent circuit are calculated in time domain in order to simulate the voltage and current waveforms. The aim of this research is to define peak efficiency according to coefficient variations of WPT system.

2. System Overview and Circuit Model

The main target of wireless power transfer is to store the energy within a coil in the magnetic resonance circuit and then to transfer this stored energy to another coil. The analysis of such a system presented in this paper combines CMT (Coupled Mode Theory) and the circuit theory. Although CMT is only applicable to coils with small mutual coupling and large coupling distance, circuit theory is applicable near-field non-radiative resonating circuits [14]. System analysis in terms of electrical circuit theory is essential since magnetic resonance coupling involves creating LC resonance. To derive the analytical equations of the input impedance and transferred power, a series-series (SS) circuit model of the WPT systems is presented as shown in Fig. 1. In this circuit, magnetic coupling is formed between transmitter coil L_1 and receiver coil L_2. k is the magnetic coupling factor ($0 < k < 1$) whereas L_m is the mutual inductance



(a) (b)

Fig. 1. Equivalent circuit of WTP system. (a) SS circuit model of wireless power transfer system. (b) Equivalent circuit of T-type coupling

In this figure, the power source V_1 is an ideal sinusoidal source. The power delivered to load impedance Z_{load} is maximized when the impedance of inductors L_1 and L_2 are cancelled by the impedance of the capacitors C_1 and C_2. The equivalent impedance given in [15] is calculated based on the equivalent circuit.

$$Z_{eq} = R + \frac{1}{j\omega C_1} + j(L_1 - L_m)\omega + \left(\frac{1}{jL_m\omega} + \frac{1}{j(L_2 - L_m)\omega + (1/j\omega C_2) + Z_0 + R}\right)^{-1} \quad (1)$$

Although the general way of detecting efficiency of transmission in an electromagnetic network is to calculate scattering parameters by using two-port network theory via network analyzer, main target of this study is to determine the efficiency by using electrical correlations. Since voltage and current are electrical quantities, the voltage equation can be written such that electrical efficiency could be calculated. L_1 and L_2 are self-inductances of resonators and the mutual inductance is $L_m = k\sqrt{L_1 L_2}$. Assuming $C = C_1 = C_2$ in the resonance coupling system the efficiency can be defined by Eq. (2) [16].

$$\eta = \left[\frac{jL_m\omega}{jL_2\omega + \frac{1}{j\omega C} + Z_0 + R_2}\right]^2 \frac{Z_0}{\left(R + jL_1\omega + \frac{1}{j\omega C} + \frac{L_m^2\omega^2}{jL_2\omega + \frac{1}{j\omega C} + Z_0 + R_2}\right)} \quad (2)$$

By running system simulations, analysis were made to calculate parameters like current and voltage as a function of frequency at various air gap values and load conditions.

It was observed that the distance between the coils responsible for power transfer should be kept at an optimum value so that the power transfer ratio will not be degraded by further increase of the air gap value. This problem could be solved by optimizing the relation between frequency and quality factor.

$$Q = \sqrt{\frac{L}{C}}\frac{1}{R} = \frac{\omega_0 L}{R} \quad (3)$$

The geometric average of the quality factors of receiving (Q_r) and transmitting (Q_t) resonators determine the quality factor of the system.

$$Q = \sqrt{Q_r \cdot Q_t} \quad (4)$$

In a highly resonant wireless power transfer system, the

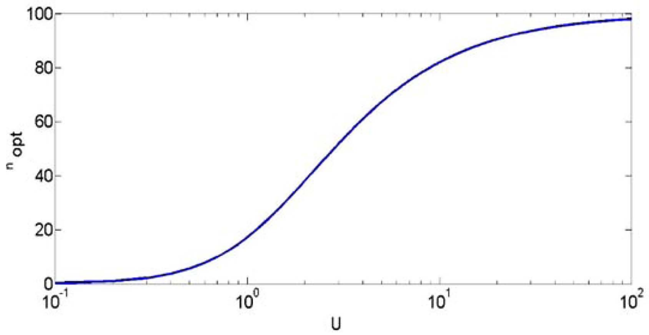


Fig. 2. (Color online) Maximum transmission efficiency plotted along figure-of-merit.

resonator system should have a high quality factor to accomplish efficient power transfer. High quality factor electromagnetic resonator is normally made with conductive material having lower losses. In the meantime, resonance frequency range is kept narrow [17].

There is a distinct coupling factor for different values of the air gap. The multiplication of the coupling factor and the quality factor reveals the figure of merit of the system which defines the boundary of the system efficiency [5].

$$U = k\sqrt{Q_r \cdot Q_t} \tag{5}$$

$$\eta_{opt} = \frac{U^2}{(1+\sqrt{1+U^2})^2} \tag{6}$$

The efficiency depending on U is illustrated in Fig. 2.

Systems with large values of U make it possible to transfer energy efficiently. On the condition that the air gap is constant, the quality factor will become high within the high values of figure of merit. If high quality factor is to be determined in the series resonance circuit, high voltage on the capacitor must be taken into consideration. Provided that the quality factor is below 10, power is transferred inductively causing the maximum air gap, at which the system can reach high efficiency boundary, decreases. As a result, in order to enable wireless power transfer for longer distances, having high value of the quality factor is essential.

The drawback of the high quality factor is the fact that the peak voltage of the capacitor becomes too high. The relationship between the peak value of capacitor voltage and quality factor is shown in Eq. (7).

$$V_{cpeak} = Q\frac{V_s}{\pi} \tag{7}$$

According to Eq. (5), figure of merit increases as quality factor increases. Yet, as it has just been explained above, quality factor is determined with respect to the maximum voltage of the capacitor that can be used.

3. Characteristic Impedance Variations and Peak Efficiency Analysis

In this section, mathematical expression of coupling coefficients are examined taking into account mutual coupling model which is presented along with a derivation of key system identifiers such as transmission distance, characteristic impedance and resonance frequency. System parameters are shown in Table 1.

The voltage transfer function of the system with SS compensation topology is obtained by using the equivalent circuit.

$$G_v = \frac{jL_m\omega}{jL_2\omega + \left(\frac{1}{j\omega C_2}\right) + Z_0 + R_2} \cdot \frac{Z_0}{R_1 + \left(\frac{1}{j\omega C_1}\right) + j(L_1 - L_m)\omega + \frac{(jL_m\omega)\left(j(L_1-L_m)\omega + \left(\frac{1}{j\omega C_2}\right) + Z_0 + R_2\right)}{jL_2\omega + \left(\frac{1}{j\omega C_2}\right) + Z_0 + R_2}} \tag{8}$$

The denominator of Eq. (8) shows that two-coil resonator system has three resonance frequency and G_v becomes maximum at two of them. Since Z_{eq} is maximum in the third resonance frequency point, implied power becomes lower. The characteristics of voltage gain function G_v vs. frequency is shown in Fig. 3.

As seen in Fig. 3, G_v rises as Z_0 increases and exceeds 1

Table 1. Specifications of Proposed System.

Parameter	Value
L_1	1010.5 nH
L_2	1032.8 nH
L_m (for 15 cm)	80.5 nH
k (for 15 cm)	0.0788
Q_t	409.62
Q_r	17.26
$C_1 = C_2$	124 pF
L_m (critical)	55.837 nH
k (critical)	0.0554
Q	84.09
U (for 15 cm)	6.6743
η_{opt} (for 15 cm)	%74.2

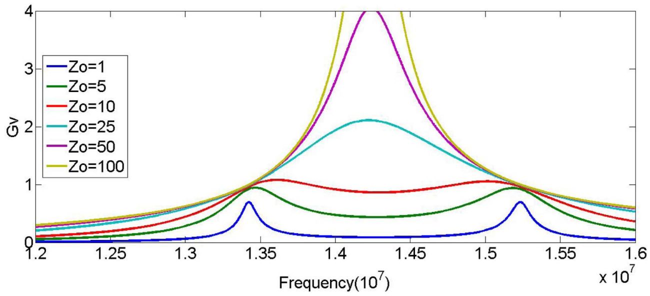


Fig. 3. (Color online) Illustration of voltage transfer function versus normalized frequency.

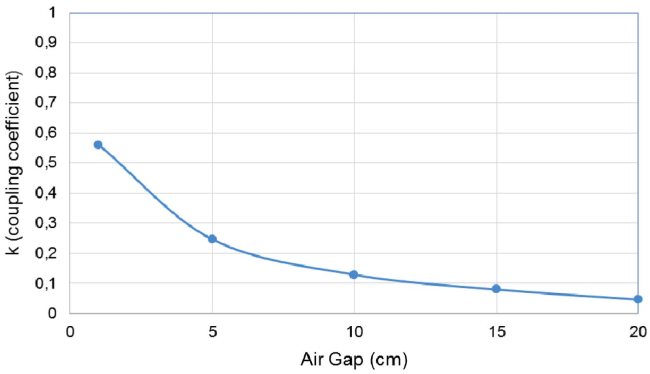


Fig. 4. (Color online) Variation of magnetic coupling factor with air gap of the system.

after a certain characteristic impedance value.

It is essential to carefully analyze the trend of coupling as variation. Fig. 4 illustrates the variation of coupling coefficient vs. air gap. From this figure, it is possible to conclude that the longer the distance between transmitter and receiver becomes, the lower the coupling coefficient *k* becomes smaller. When the distance between the resonators is getting shorter, *k* increases that means a higher magnitude of coupling value.

Considering Eq. (2) and the variations of *k* depending on the air gap, Fig. 5 illustrates the changes in the efficiency with respect to *k* and frequency.

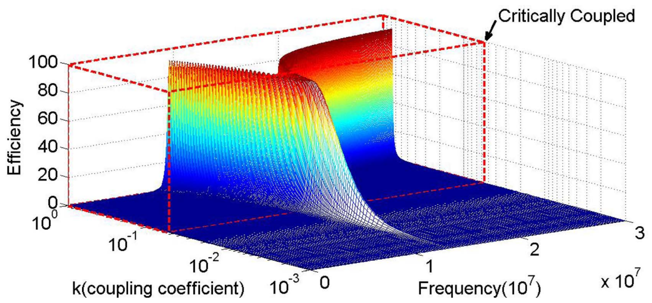


Fig. 5. (Color online) Function of efficiency according to *k* and frequency for Z_0 = 5 Ω.

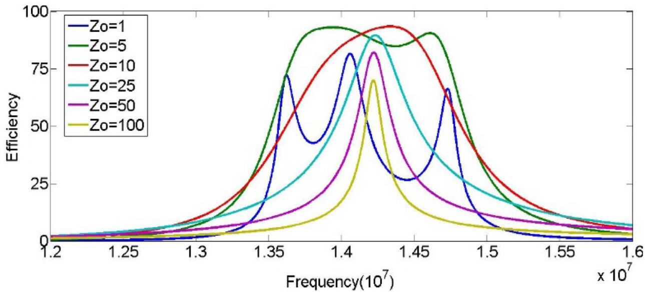


Fig. 6. (Color online) Illustration of efficiency versus normalized frequency.

When *k* is equal to 0.0778, obtained from Fig. 4 for the air gap value of 15 cm, is placed in Fig. 5; it can be seen that the operating point is within the zone of strongly coupled regime. The coupling factor value, at which the resonance frequency becomes singular and the efficiency begins to decrease, is called $k_{critical}$ and calculated with Eq. (9).

$$k_{critical} = \sqrt{\frac{Z_0^2 - R^2}{\omega_0^2 L_1 L_2}} \tag{9}$$

As illustrated in Table 1, $k_{critical}$ is calculated as 0.0554 for this study. By using Neumann Formula, the air gap corresponding to critical distance is calculated as 18.9 cm [11].

For varying values of Z_0, the change in the efficiency with respect to the frequency is illustrated in Fig. 6.

When Fig. 3 and Fig. 6 are analyzed together, it is seen that G_v and the efficiency is maximum at resonance frequency. G_v exceeds 1 within the characteristic impedance values of 25, 50, and 100 Ω however the efficiency decreases at these points. As a result, the characteristic impedance values of 1, 5 and 10 Ω are found to be appropriate for transferring power with peak efficiency. Since Z_{eq}/Z_0 considerably decreases when the characteristic impedance is 1 Ω, power cannot be transferred with peak efficiency. Provided that the characteristic impedance is selected as 10 Ω, the operation of the system in a strongly coupled regime will be difficult with the increase of $k_{critical}$. When the characteristic impedance is selected as 5 Ω, $k_{critical}$ is lower than *k*, therefore, operating in the strongly coupled zone will be easy and the efficiency will increase.

System efficiency is analyzed depending on different air gap values for various characteristic impedances in strongly coupled magnetic resonance regime. The non-radiative fields allow efficient power exchange between source and device resonators. The resonant frequencies change from three points to one point depending on the

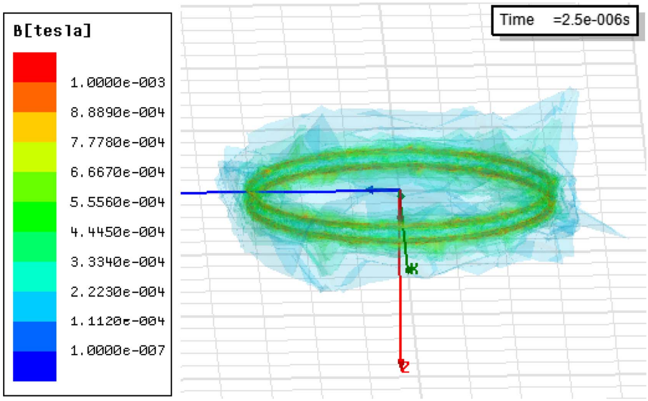


Fig. 7. (Color online) Real geometry of the WPT system and magnetic field distribution in the vicinity of the conventional circular resonant loop for air gap = 1 cm.

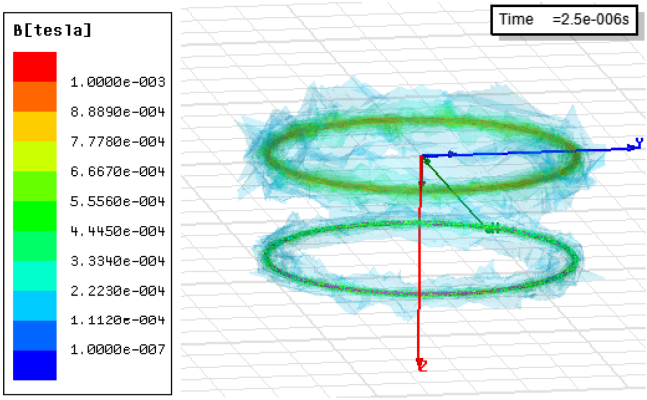


Fig. 9. (Color online) Real geometry of the WPT system and magnetic field distribution in the vicinity of the conventional circular resonant loop for air gap = 10 cm.

length of the air gap. The triple resonance frequency region occurs at low impedance and short range. As the air gap distance and impedance increase, one resonance region occurs. At this operation range, the efficiency falls. The critical transmission efficiency would be same with peak efficiency of over coupled range.

Equivalent circuit is solved by Ansys® Maxwell 3D software. The procedure to calculate the parameters of the equivalent circuit has been carried out in Ansys® Maxwell 3D software platform as well. Figure 7 shows the 3-dimensional magnetic field distribution for both receiver (upper coil) and transmitter coils which are 1 cm away from each other. As it is depicted in Fig. 7, the sample time is at 2.5 μs and it is seen from figure that the coupling has been established and the transmission has been started.

Since the sample time depends on transmission frequency and consequently the coupling coefficient and the air gap, 3-dimensional magnetic field distribution changes in various circular resonant loops for different air gap values. Figure 8 shows the 3-dimensional magnetic field distribution for both receiver (upper coil) and transmitter coils which are 5 cm away from each other. As it is depicted in Fig. 8, the sample time is at 2.5 μs and it is seen from figure the coupling has been established and the transmission has been started.

Figure 9 and Fig. 10 show the 3-dimensional magnetic field distribution for both receiver (upper coil) and transmitter coils which are 10 cm and 15 cm away from each other respectively. For Fig. 9 the sample time is at 2.5 μs and for Fig. 10 it is 2.44 μs. Since the air gap value of 15 cm is smaller than the critical coupling distance value of 18.9 cm, the peak efficiency of wireless power transfer could be reached at this position.

Figure 11 shows magnetic field distribution in the vicinity of the conventional circular resonant loop for 20

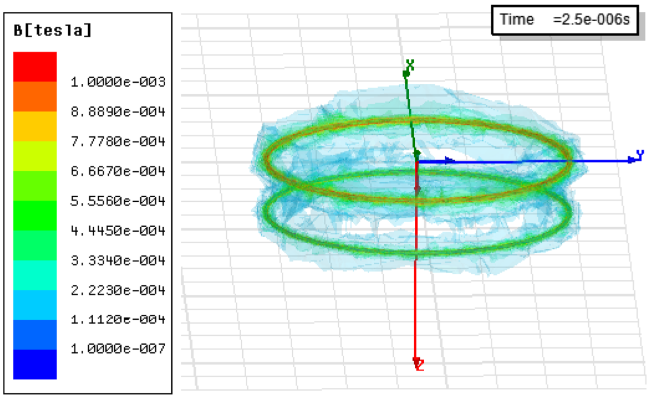


Fig. 8. (Color online) Real geometry of the WPT system and magnetic field distribution in the vicinity of the conventional circular resonant loop for air gap = 5 cm.

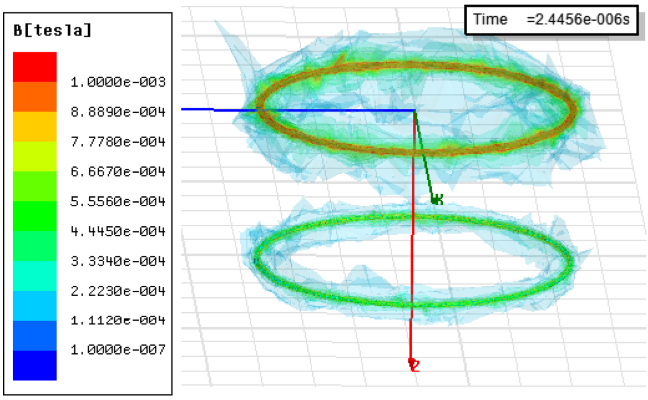


Fig. 10. (Color online) Real geometry of the WPT system and magnetic field distribution in the vicinity of the conventional circular resonant loop for air gap = 15 cm.

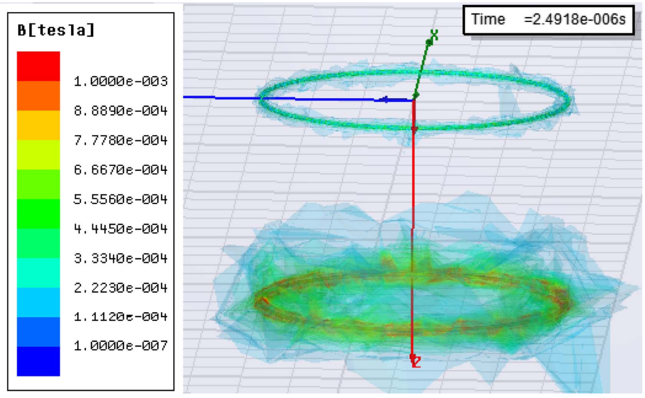


Fig. 11. (Color online) Real geometry of the WPT system and magnetic field distribution in the vicinity of the conventional circular resonant loop for air gap = 20 cm.

cm. Above critical distance of 18.9 cm, coils would behave as air-core inductances and receiver coil could drain no more energy since the coupling is so weak. So, this time the transmitter coil is exposed high amount of energy that is not being transferred.

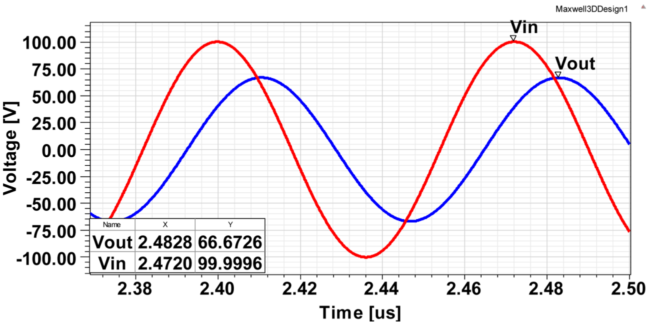


Fig. 12. (Color online) Airgap 15 cm, characteristic impedance 5 Ω at 13.56 MHz input voltage (V_{in} red line) and device voltage waveforms (V_{out} blue line).

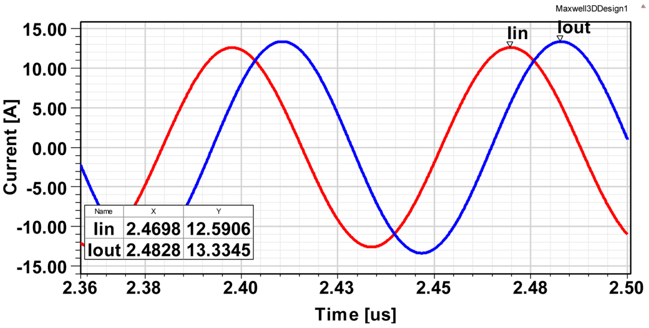


Fig. 13. (Color online) Airgap 15 cm, characteristic impedance 5 Ω at 13.56 MHz input current (I_{in} red line) and device input waveforms (I_{out} blue line).

L_m will rapidly decrease especially during the cases where both of the resonators are not aligned [15]. Therefore, the air gap of 18.9 cm, which is calculated as critical coupling distance in the aligned state, will become much smaller when the resonators are not aligned. In this study, the maximum air gap that can be reached is chosen as 15 cm and the variations of input - output voltage and current are illustrated in Fig. 12 and Fig. 13 respectively.

The calculated root means square of transmitter current is 8.90 A and the receiver current is 9.43 A for a supply voltage of 70.71 V and the receiver voltage of 47.14 V. While input power is 629.3 W, the amount of output power is 444.5 W and approximately 184.8 W is dissipated as loss. So, the transmitter requires 0.415 W (an overhead loss) of additional input power per 1W of power transmission to the receiver to have a peak efficiency of 70.63 %.

4. Conclusion

The aim of this research is to define the peak efficiency according to coefficient variations of WPT system. Ansys® Maxwell 3D software is utilized to solve equivalent circuit and also to calculate mutual inductance and characteristic impedance according to air gap variations.

It is concluded that equivalent circuit analysis by means of numerical computing is proper to obtain the voltage and current waveforms and as well as the amount of transferred power. Correspondingly, peak efficiency at a different distance range can be calculated by using the electrical relations. Efficiency results with respect to load variations show that there are triple resonance frequency regions as well as singular resonance region. The point in which singular resonance region appears is the critical coupling point. Up until reaching this point, the rate of change in efficiency is lower. After reaching this point, efficiency decreases drastically. As the resonant frequencies change from three points to one point depending on the length of the air gap, triple resonance frequency region occurs at low impedance and short range. While the airgap distance and impedance increase, singular resonance region occurs and efficiency falls.

Acknowledgement

This research was supported by Yildiz Technical University Scientific Research Projects Coordinatorship grant (2016-04-02-DOP03).

References

[1] C. Wang, G. A. Covic, and O. H. Stielau, IEEE Trans. Ind. Electron. **51**, 148 (2004).
[2] A. Sahai and D. Graham, IEEE International Conference

on Space Optical Systems and Applications (ICSOS) **1**, 164 (2011).

[3] Z. N. Low, J. J. Casanova, P. H. Maier, and J. A. Taylor, IEEE Trans. Ind. Electron. **57**, 1478 (2010).

[4] A. P. Sample, B. H. Waters, S. T. Wisdom, and J. R. Smith, Proceedings of the IEEE **101**, 1343 (2013).

[5] A. P. Sample, D. A. Mayer, and J. R. Smith, IEEE Trans. Ind. Electron. **58**, 544 (2011).

[6] A. Kurs, A. Karalis, R. Mofatt, J. D Joannopoulos, P. Fisher, and M. Soljacic, Science **317**, 83 (2007).

[7] Y. Zhuo, Y. Qingxin, C. Haiyan, Z. Chao, and X. Guizhi, in Proc. Int. Conf. on Elect. and Contr. Engineering **1**, 3886 (2010).

[8] W. Wei, Y. Narusue, Y. Kawahara, N. Kobayashi. H. Fukuda, and T. Tsukagoshi, in Proc. IEICE Tech. Committee Meeting 1 (2012).

[9] C. Chih-Jung, C. Tah-Hsiung, L. Chih-Lung, and J. Zeui-Chown, IEEE Trans. Circ. and Sys. II: Express Briefs **57**, 536 (2010).

[10] Y. H. Kim, S. Y. Kang, M. L. Lee, B. G. Yu, and T. Zyung, in Proc. Int. Conf. on Computability and Power Electron. 426 (2009).

[11] T. Imura and Y. Hori, IEEE Trans. Ind. Electron. **58**, 4746 (2011).

[12] R. A. Salasa and J. Pleite, J. Appl. Phys. **107**, 1 (2010).

[13] J. H. Park and S. W. Kim, J. Magn. **18**, 105 (2013).

[14] M. Kiani and M. Ghovanloo, IEEE Trans. Circuits Syst. I: Regul. Pap. **59**, 2065 (2012).

[15] A. Agcal, N. Bekiroglu, and S. Ozcira, J. Magn. **21**, 652 (2016).

[16] A. Agcal, N. Bekiroglu, and S. Ozcira, J. Magn. **20**, 57 (2015).

[17] A. Agcal, S. Ozcira, and N. Bekiroglu. Wireless Power Transfer by Using Magnetically Coupled Resonators, Wireless Power Transfer - Fundamentals and Technologies, Dr. Eugen Coca (Editor), Ch. 3, 46 InTech Croatia (2016).