Simultaneous Effects of Slip and Wall Stretching/Shrinking on Radiative Flow of Magneto Nanofluid Through Porous Medium

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Effects of the uniform magnetic field on aqueous magneto-Nanofluid confined in a porous domain with wall stretching/shrinking non-linearly is analyzed via this communication. The problem is modeled using continuity, momentum and energy equation along with linear thermal radiation. The effects of physical quantities are observed for Cu, Al₂O₃, TiO₂ and Ag particles in water. The coupled boundary layer PDE's are reduced into the system of ODEs by utilizing similarity transformation and solved using shooting and Runga-Kutta fourth order technique. Stability of the obtained results are also analyzed. The results are displayed through graphs. It is observed that the momentum boundary layer is thicker when silver particles are introduced in water. Whereas, temperature profile has the minimum value for silver nanoparticles and maximum for Titanium dioxide. Also, in case of shrinking sheet dual solutions are obtained along with smallest Eigen values.

Keywords : magnetic Nanofluid, second order slip boundary condition, non-linear stretching/shrinking sheet, porous medium, thermal radiation

1. Introduction

Modern developments have revolutionized into novel engineering nanotechnology, which led to the progress of a new sort of heat transfer fluid called Nanofluid. Nanofluids are used widely in industrial applications including nuclear reactors, transportation industry, engine cooling, hyperthermia, biomedicine, vehicle technology, solar water heating and heat exchanger. Nanofluids gain rapid attention to the researcher for both theoretical and experimental point of view [1-3]. Choi et al. [4] was the first one who gave the idea of Nanofluid comprising of solid nanoparticles with size approximately 10 nm. Nano-Fluids are dilute suspended metallic particles (gold, titanium, copper, iron and their oxides) into conventional fluids like water, ethylene glycol, oil and lubricants [5]. Eastman et al. [6] have verified experimentally that the thermal conductivity of the nanoparticles is greater than conventional fluids. Li et al. [7] demonstrated that for a lower concentration of nanoparticles typically (< 5%), the thermal conductivity of the conventional fluid is improved by 10-20%. Sheikholeslami et al. [8] sees the influence of magnetic field on Nanofluid. Baby and Ramaprabhu [9] have studied the Nanofluids comprising graphene for thermal conductivity and heat transfer. Bachok et al. [10] inspected the Nanofluid flow over a moving semi-infinite flat surface with the hypothesis that the plate moves in the same or reverse directions to free stream. Makinde and Aziz [11] have demonstrated the influence of convective boundary condition and heat transport flow over a stretchable surface fill with Nanofluid. Zeeshan et al. [12] discussed the effect of magnetization of the ferromagnetic nanoparticle in water. The effects of aggregation of nanoparticles are discussed by Ellahi et al. [13]. The entropy generation due to the presence of nanoparticles in the fluid over a revolving plate is discussed by Rashidi et al. [14].

The discussion regarding the magnetization effect has imperative presentations in the field of physics, chemistry, industrial and engineering stuff, like MHD generators, bearings, pumps, and boundary layer control are exaggerated by the association among the magnetic field and
electrically conducting fluid [15, 16]. Ganga et al. [17] investigated the viscous and Ohmic dissipations on magnetohydrodynamic radiative Nanofluid flow over a vertical plate along with internal heat generation or absorption. Some recent studies on for Nanofluid flowing through uniform and non-uniform magnetic field are listed in [18-22].

Furthermore, in micro-electro-mechanical systems, the fluid flow behaviour does not obey the no-slip boundary condition. So, slip conditions are necessary in order to illuminate the fluid flow. When the fluids composed of distinct particles like suspensions, foams, polymer solutions and emulsions, partial slip may take place across a moving surface. Slip flow has tremendous applications like the smoothening of internal cavities and artificial heart shells. Turkylmazoglu [23] analyzed MHD second-order slip model and heat transport flow over a permeable sheet is observed. A uniform magnetic field is applied aqueous medium flowing over a non-linearly stretching heart shells. Turkyilmazoglu [23] analyzed MHD second-order slip model and heat transport flow over a permeable sheet is applied. The base fluid is considered as water. The continuity, Naiver-Stokes and heat equations for incompressible electrically conducting viscous magneto-Nanofluid over a thin sheet is employed for modeling the problem. The magnetic field of strength \( B_0 \) normal to the sheet is applied. The base fluid is considered as water. Covering boundary-layer equations are:

\[
\begin{align*}
    u_x & + v_y = 0 \quad (1) \\
    uu_x + vu_y = \frac{\sigma B^2(x)}{\rho_{nf}} u - \frac{\nu_{nf}}{k^*} u + v_{nf} & \quad (2) \\
    (\rho c_p)_{nf} (u T_x + v T_y) & = \alpha_{nf} T_{xx} - q_y \quad (3)
\end{align*}
\]

The associative boundary equations for velocity and temperature fields are

\[
\begin{align*}
    u &= u_w + u_{slip} , \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \quad (4) \\
    u &= 0, \quad T \rightarrow T_{\infty} \quad \text{at} \quad y \rightarrow \infty, \quad (5)
\end{align*}
\]

Where \( u_w = c_s \left( \frac{2}{k^*} \right) \), \( u \) and \( v \) are components of velocity in the direction of coordinate axis and \( T \) is the Nanofluid temperature. The symbols \( v_{nf} \), \( \rho_{nf} \) and \( \sigma \) represents kinematic viscosity, density, and electrical conductivity of Nanofluids \( (c_{nf})_{nf}, \alpha_{nf} \) are heat capacity, thermal conductivity, and \( q_r \) is the radiative heat flux and given by Quinn Brewster using Roseland approximation,

\[
q_r = \left( \frac{4 \sigma^*}{3k^*} \right) (4TT_w^2 - 3T_s^2)
\]

Using the below expression of Nanofluid

\[
\begin{align*}
    \rho_{nf} &= (1 - \phi) \rho_\gamma + \phi \rho_s, \quad \mu_{nf} = \frac{\mu_\gamma}{(1 - \phi)^{2.5}}, \\
    \alpha_{nf} &= (\alpha_s + 2 \alpha_r) - 2(\phi \alpha_s - \alpha_r), \\
    \frac{\alpha_{nf}}{\alpha_f} &= (\alpha_f + 2 \alpha_s) + 2(\phi \alpha_s - \alpha_r), \\
    (\rho c_p)_{nf} &= (1 - \phi)(\rho c_p)_s + \phi (\rho c_p)_s
\end{align*}
\]

with stream function transformation, \( u = \psi_x \) and \( v = -\psi_y \), where

\[
\eta(x, y) = \left( \frac{c(n + 1)x^{n-1}}{2v} \right)^\frac{1}{2}, \quad \psi(x, y) = \left( \frac{2v cx^{n-1}}{(n + 1)} \right)^\frac{1}{2} f(\eta),
\]

\[
T = T_w + (T_u + T_v) g(\eta) \quad \text{and} \quad T_w - T_u = (T - T_w) x^{2n}
\]

we get

\[
(1 - \phi)^{-2.5} f''' + (1 - \phi) + \phi \frac{\partial \rho_x}{\rho_f} (ff'' - \beta f'^2)
\]

\[
- \left( M + \frac{(1 - \phi)^{-2.5}}{k_p} \right) f'' = 0 \quad (6)
\]

\[
\left( \frac{\alpha_{nf}}{\alpha_f} + R \right) g'' + Pr (1 - \phi) + \phi \frac{(\rho c_p)_s}{(\rho c_p)_s} (g'f - f'g) = 0 \quad (7)
\]

The boundary condition will reduce into:

### 2. Mathematical Formulation

#### 2.1. Stretching/Shrinking sheet

The continuity, Naiver-Stokes and heat equations for incompressible electrically conducting viscous magnetofluid are presented on velocity and temperature profile in form of graphs. Finally, some important observations are concluded.
\( f(0) = 0, \ f'(0) = \lambda + \gamma f''(0) + \delta f'''(0), \ g(0) = 1 \) at \( \eta = 0 \) \( f'(\infty) \to 0, \ g(\infty) \to 0 \) at \( \eta \to \infty \)

\( k' \) is the porosity variable and it is equal to \( k'x^{1-n} \), \( u_u = cx^2 \), here \( c \) is the rate of stretching/shrinking and

\[ u_{u0} = \frac{2}{n+1} \left( \frac{16 \sigma^{*} R}{3(c_p \mu) k^*} \right) \]

\( M = \frac{2 \sigma B_0^2}{c_p(n+1)} \), \( 1 = \frac{2 v_f}{(n+1)c_k^*} \)

The boundary condition will reduce to:

\[ f(0) = 0, \ f'(0) = \lambda + \gamma f''(0) + \delta f'''(0), \ g(0) = 1 \]

at \( \eta = 0 \)

\[ f'(\infty) \to 0, \ g(\infty) \to 0 \] at \( \eta \to \infty \)

The power index \( \beta = 0 \), the parameter of magnetic field

is \( M^* = \frac{\sigma B_0^2}{c_p \rho_f} \), the parameter of permeability is \( \frac{1}{k_p} = \frac{v_f}{c_p \ k^*} \)

the Prandtl number is \( Pr = \frac{(c_p \mu)}{\alpha_f} \), and Radiation number

is \( R^* = \frac{16 \sigma^* T_0^*}{3(c_p \mu) k^*} \), respectively.

### 2.2. Stability Analysis

In order to analysis stability, we consider unsteady momentum and temperature profile as defined by [28],

\[ u_x + uu_x + vv_y = v_n u_y - \frac{\sigma B^* (x)}{\rho_n} u - \frac{\nu_f}{k^* (x)} u \]

\[ T_x + (\rho c_p)_n (u T_x + v T_y) = \alpha_n T_{xx} - q_{xy} \]

With continuity equation together with unsteady similarity transformation,

\[ \eta(x, y) = \left( \frac{2(n+1)x^{n-1}}{v} \right)^{\frac{1}{2}} y, \ \psi(x, y) = \left( \frac{2 v c x^{n-1}}{(n+1)} \right)^{\frac{1}{2}} f(\eta, \tau) \]

For \( f(0) = 0, \ f'(0) = \lambda + \gamma f''(0) + \delta f'''(0), \ g(0) = 1 \) at \( \eta = 0 \) \( f'(\infty) \to 0, \ g(\infty) \to 0 \) at \( \eta \to \infty \)

\[ T = T_w + (T_u - T_w) g(\eta, \tau) \]

where the subscript \( \eta \) denotes partial derivative with respect to \( \eta \) and \( \tau \) is the dimensionless time \( c t \). We seek the boundary conditions as eq. (4) and (5)

In terms of ordered pair \( (\eta, \tau) \)

\[ (1 - \phi)^{-\frac{5}{2}} f_{qqq}'' + (1 - \phi + \phi \frac{\rho_f}{\rho_f}) (f f'' - \beta f^2) - \left( M + \frac{1}{k_p} \frac{1}{k_p} \right) f_{q} - (1 - \phi + \phi \frac{\rho_f}{\rho_f}) f_{q} = 0 \]

\[ \left( \frac{\alpha_f + R}{\alpha_f} \right) \phi + Pr \left( (1 - \phi) + \phi \frac{\rho c_p}{\rho c_p} \right) (g f - f g) \]

\[ - \left( (1 - \phi) + \phi \frac{\rho_f}{\rho_f} \right) g_{\tau} = 0 \]

The boundary condition will reduce into:

\[ f(0, \tau) = 0, \ f'(0, \tau) = \lambda + \gamma f''(0, \tau) + \delta f'''(0, \tau), \ g(0, \tau) = 1 \]

\[ f'(\infty, \tau) \to 0, \ g(\infty, \tau) \to 0 \]

To show the stability of the solution, we set \( f = f_0(\eta) \) and \( g = g_0(\eta) \), which satisfying the boundary value problem (6-9), we suppose that

\[ f(\eta, \tau) = f_0(\eta) + e^{\sigma \tau} F(\eta, \tau) \]

\[ g(\eta, \tau) = g_0(\eta) + e^{\sigma \tau} G(\eta, \tau) \]

where \( \sigma \) is an unknown eigenvalue parameter, while \( F(\eta, \tau) \) and \( G(\eta, \tau) \) are small relative to \( f_0(\eta) \) and \( g_0(\eta) \). Substituting (20) and (21) into (16-19), and after linearizing problem, we obtain the following equations

\[ (1 - \phi)^{-\frac{5}{2}} F_{qqq}'' + (1 - \phi + \phi \frac{\rho_f}{\rho_f}) (f f'' - \beta f^2) - \left( M + \frac{1}{k_p} \frac{1}{k_p} \right) f_{q} - (1 - \phi + \phi \frac{\rho_f}{\rho_f}) f_{q} = 0 \]

\[ \left( \frac{\alpha_f + R}{\alpha_f} \right) G_{q} + Pr \left( (1 - \phi) + \phi \frac{\rho c_p}{\rho c_p} \right) (g f - f g) - (1 - \phi) + \phi \frac{\rho_f}{\rho_f} \right) g_{\tau} = 0 \]

Subject to boundary condition

\[ F(0, \tau) = 0, \ F'(0, \tau) = \lambda + \gamma F''(0, \tau) + \delta F'''(0, \tau), \ G(0, \tau) = 1 \]

\[ F'(\infty, \tau) \to 0, \ G(\infty, \tau) \to 0 \]

Now, we investigate the stability of the steady Nano-fluid flow and temperature transfer solution \( f_0(\eta) \) and \( g_0(\eta) \) by setting \( \tau = 0 \), and hence \( F = F_0(\eta) \) and \( G = G_0(\eta) \)
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in (22-23) to identify initial growth or decay of the solution (20) and (21). To test our numerical procedure we have to solve the following linear eigenvalue problem:

\[
(1-\phi)^{-2.5} F_0'''' + \left(1-\phi + \phi \frac{\rho f}{\rho_j} \right) (f_0 F_0'' + F_0 f_0'' - 2\beta_0' F_0') - M_{j} + \left(1-\phi - \frac{\alpha f}{\alpha_j} \right) \left(1-\phi + \phi \frac{\rho f}{\rho_j} \right) (\sigma F_0') = 0
\]

\(26\)

\[
\left(\frac{\alpha f}{\alpha_j} + R \right) G_0'' + Pr \left(1-\phi + \phi \frac{\rho c_p}{\rho_p} \right) (f_0 G_0' + F_0 g_0' - f_0' G_0 - F_0' g_0') + \left(1-\phi + \phi \frac{\rho f}{\rho_j} \right) (\sigma G_0) = 0
\]

\(27\)

Subject to boundary condition

\[F_0(0) = 0, \quad G_0'(0) = \lambda + \gamma F_0''(0) + \delta F_0'''(0), \quad G_0(0) = 1\]

\[F_0'(\infty) \to 0, \quad G_0(\infty) \to 0\]

\(28\)

Using the numerical values of \(F(\eta)\) and \(G(\eta)\), Eqs. (26-28) are obtained. Solutions give an infinite set of eigenvalues \(1 < \sigma 2 < \sigma 3 < \cdots\), if the smallest eigen-value \(\sigma 1\) is positive, then there is an initial decay of disturbances and the flow is stable. On the other hand when \(\sigma 1\) is negative, there is an initial growth of disturbances and the flow is unstable.

The physical parameters on boundary flow problem are the local skin friction coefficient \(C_f\) and Nusselt number \(Nu\), which are formulated as

\[C_f = 2(1-\phi)^{-2.5} Re_x f''(0),\]

\[Nu = -2 \frac{\alpha f}{\alpha_j} Re_x g'(0),\]

where \(Re_x = \frac{u_w x}{\nu (n + 1)}\) is local Reynold number, for \(\phi = 1\), we get \(C_f\) and \(Nu\) fluid flow without nanoparticle.

3. Figures and Tables

In this segment, numerical outcomes for velocity and temperature field are elaborated through graphs. The computations are performed to study the impact of numerous physical flow parameters that describe the flow characteristics.

We have taken four kinds of nanoparticles i.e., Copper (Cu), Titanium oxide (TiO\(_2\)), Aluminum dioxide (Al\(_2\)O\(_3\)), silver (Ag) with water as a base fluid. Table 1 shows the thermo physical properties of water and the elements Cu, Al\(_2\)O\(_3\), Ag and TiO\(_2\). During the stability analysis, it is

<table>
<thead>
<tr>
<th>Materials</th>
<th>(\rho) (kg/m(^3))</th>
<th>(c_p) (J/kgK)</th>
<th>(\alpha) (W/m(\cdot)K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Water (H(_2)O)</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>8933</td>
<td>385</td>
<td>401</td>
</tr>
<tr>
<td>Alumina (Al(_2)O(_3))</td>
<td>3970</td>
<td>765</td>
<td>40</td>
</tr>
<tr>
<td>Titanium oxide (TiO(_2))</td>
<td>4250</td>
<td>686.2</td>
<td>8.9538</td>
</tr>
<tr>
<td>Silver (Ag)</td>
<td>10500</td>
<td>234</td>
<td>429</td>
</tr>
</tbody>
</table>

Table 1. Material properties.

Fig. 1. (Color online) Velocities distribution \(f'(\eta)\).

Fig. 2. (Color online) Temperature distribution \(g(\eta)\).
Fig. 3. (Color online) (a-d) Velocity of nano fluids for various values of $\beta$.

Fig. 4. (Color online) (a-d) Velocity of Nanofluid for numerous values of $\gamma$. 

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Fig. 5. (Color online) (a-d) Velocity of Nanofluid for numerous values of $M$.

Fig. 6. (Color online) (a-d) Temperature of Nanofluid for various values of $Pr$. 
seen that for $\lambda > 0$ (stretching sheet) the solution is stable. Whereas, $\lambda < 0$ (shrinking sheet) for some values of parameter dual solution emerges. Smallest eigenvalues are obtained for both the solutions it is observed that the lower branch solutions have negative smallest Eigenvalue, and upper branch has positive eigenvalues. So, the upper branch of the solution is physically stable. Smallest eigenvalues calculated for different values of $\lambda < 0$ for all nanoparticles are shown in Table 2. Figure 1 shows the velocity distribution $f'(\eta)$ of nano-particles. It is noted that silver gain maximum velocity as compared to other nanoparticles. Figure 2 shows temperature distribution $g(\eta)$ in which silver has minimum temperature distribution as compared to others Nano-particles. Figure 3(a-d) illustrate that the velocity profile of silver, copper, Aluminum di oxide and Titanium with various values of power index $\beta$. It is noted that when values of $\beta$ increases, the velocity of Nanofluid also increased. Figure 4(a-d) shows the velocity profile of Nanofluid with different values of $\gamma$. It shows that when values of $\gamma$ increases, the velocity of Nanofluid is also increased. Figure 5(a-d) shows the velocity profile of Nanofluid with different values of magnetic parameter $M$. It shows that when values of $M$ increases, the velocity profile of Nanofluid decreases. Figure 6(a-d) pointed out that temperature for various value of $Pr$. It is found that as the $Pr$ increases, the temperature is also increased. Figure 7(a-d) shows the consequence of radiation parameter $R$ on temperature distribution. It shows that when the values of $R$ is increased, the temperature is also increasing.

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Simultaneous Effects of Slip and Wall Stretching/Shrinking on Radiative Flow—the Effect of Wall Absorption and Emissivity

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