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A New Family of Unsteady Boundary layer Flow over a Magnetized Plate

Adeel Ahmad* and Razi Khan

COMSATS University Islamabad

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This article investigates the unsteady flow of a viscous fluid over a magnetized moving plate emerging from a moving slot. This new family of unsteady boundary layer flow behaves similar to stretching and shrinking sheet problems depending upon the motion of the slot. The governing partial differential equations are reduced to correct similar form using the Blasius-Rayleigh-Stokes variable. Effects of moving slot and magnetic field on important physical quantities are examined. An interesting observation was the influence of magnetic field on the existence of dual solution for the range of moving slot parameter.

Keywords : unsteady flow, magnetized plate, Blasius-Rayleigh-Stokes variable, numerical solution, dual solution

1. Introduction

The phenomenon of drag reduction (Singh, 2004), which prevents the loss of mechanical energy, has been a topic of intensive research. Various methods have been proposed to reduce the drag in physical systems which include adding polymers in base fluid (Gyr and Bewersdorff, Kluwer, 1995), magnetic fields (Shatrov, Gerbeth 2007) and flexible walls. Magnetic field is a useful agent for drag reduction and flow control in electrically conducting fluids. Varying the orientation, geometry and time dependence of externally applied magnetic field are some of the possible ways to manipulate the electrically conducting fluid flow. In the flow of electrically conducting fluids with high conductivity, induced magnetic field plays a vital role in flow stability and drag reduction. Greenspan and Carrier (1959) was the first who investigated the flow of viscous, incompressible and electrically conducting fluid in the presence of a symmetrically oriented semi-infinite flat plate in which magnetic field assumed to be coincident with the ambient fluid velocity field. Pavlov (1974) studied a steady flow of electrically conducting fluid in the presence of uniform magnetic field due to stretching of surface and provided an accurate closed-form solution. Markin and Kamaran (2010) considered shrinking of the sheet for the unsteady magneto-hydrodynamic (MHD) flow of viscous fluid and obtained the approximate analytical solution for large magnetic field. Ahmad, Afzal and Asghar (2015) studied the unsteady boundary layer flow of an electrically conducting second grade fluid over a stretching surface and give an asymptotic analytical solution for large magnetic field. They also investigated the evolution of velocity field and skin friction with time and stability of the asymptotic solution. Some other recent investigations on the topic of MHD are (Afzal, Asghar, Ahmad, 2017), (Kumar *et al*., 2017), (Gireesha *et al*., 2017), (Prasannakumara *et al*., 2017) and (Gireesha et al., 2018).

In recent technological development it is necessary to distort the attention towards magnetized surface. Glauert (1962) first studied the MHD boundary layer in uniform flow past a magnetized plate for the small and large values of magnetic Prandtl number. Chawla (1967) studied the MHD boundary layer in uniform flow past a semi-infinite magnetize plate, and a magnetic field fluctuating about a nonzero mean in the stream direction, is applied to the plate. For some recent studies related to flow over a magnetized surface see the articles (Ashraf, Asghar, Hossain, 2010) - (Ashraf, Asghar, Hossain, 2012) and the references given therein.

The problem of an unsteady boundary-layer flow past stretching surfaces (Fang, 2008) has received much less attention for it is more difficult to analyze than the corresponding steady state problems. Todd (1977) obtained a similarity solution for unsteady flow past semi-infinite flat plate emerging from a moving slot. Depend-

*Corresponding author: Tel: +92-345-511-4884
Fax: +92-51-9247006, e-mail: adeelahmed@comsats.edu.pk

ing on the direction of motion of slot the considered problem resembles the stretching/shrinking surface flow. Fang, Zhang and Yao (2010) extended this analysis to the flow and heat transfer of viscous fluid. Considering the importance of unsteady flow of electrically conducting fluid over a moving magnetized surface and limited studies available on the topic, in this article we will use the Blasius-Rayleigh-Stokes variable to write the unsteady governing equation in correct similar form. In the proceeding section after stating the governing equations and their similar form, the effects of different parameters on the flow of the fluid and the magnetic field will be discussed in detail.

2. Mathematical Formulation

Consider a laminar two-dimensional unsteady MHD flow of an electrically conducting incompressible viscous fluid over a magnetized horizontal plate. The x-axis is along the surface and y-axis is taken normal to it. The governing boundary layer equations for the momentum and magnetic field are [Chawla (1967)]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+\frac{\mu}{\rho}\left(B_x\frac{\partial B_x}{\partial x}+B_y\frac{\partial B_x}{\partial y}\right), \tag{2}$$

$$\frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}=0, \tag{3}$$

$$\frac{\partial B_x}{\partial t}+u\frac{\partial B_x}{\partial x}+v\frac{\partial B_x}{\partial y}-B_x\frac{\partial u}{\partial x}-B_y\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 B_x}{\partial y^2}, \tag{4}$$

with the following boundary conditions

$$\begin{aligned} &u=U_w(x),\ \ v=0,\ \ B_x=B_0,\ B_y=0,\ \text{ at }\ y=0\\ &\qquad u\to 0,\ \ B_x\to 0,\ \text{ as }\ y\to\infty, \end{aligned} \tag{5}$$

where u and v represents velocity components in the x and y direction respectively, B_x and B_y are the x and y components of magnetic field (see Fig. 1 for physical interpretation), ν is the kinematic viscosity and γ is the magnetic diffusivity of the fluid. Eq. (1) and (3) are mathematical representation for the laws of conservation for mass and magnetic field also known as continuity equations. Eq. (2) is boundary layer equation for the law of conservation of momentum. Eq. (4) is known as induction or advection-diffusion equation for magnetic field and is transport equation for magnetic field in the sense if the velocity field is known subsequently it indicates the spatial and temporal evolution of magnetic field.

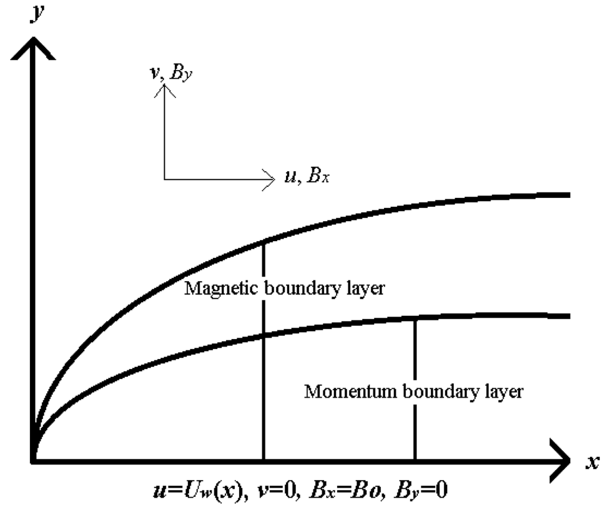


Fig. 1. Physical interpretation of the problem.

Using the stream function and components of magnetic field of the following form:

$$\psi(x,y,t)=U_w\sqrt{\cos(\alpha)\nu t+\sin(\alpha)(\nu x/U_w)}f(\eta)$$

$$B_x=B_0 g'(\eta),\ B_y=\frac{B_0\nu}{2U_w}(\eta g'-g)\sin(\alpha)/\sqrt{\cos(\alpha)\nu t+\sin(\alpha)(\nu x/U_w)}$$

identically satisfying the equations of continuity, the governing Eqs. (2) and (4) can be converted into similarity equations as follows:

$$f'''-\frac{1}{2}Sc\sin(\alpha)gg''+\frac{1}{2}\eta\cos(\alpha)f''+\frac{1}{2}\sin(\alpha)ff''=0, \tag{6}$$

$$\frac{1}{Pm}g'''+\frac{1}{2}\eta\cos(\alpha)g''+\frac{1}{2}\sin(\alpha)fg''-\frac{1}{2}\sin(\alpha)f''g=0, \tag{7}$$

where $Sc=\frac{\mu B_0^2}{\rho U_w^2}$ is the magnetic force parameter and $Pm=\frac{\nu}{\gamma}$ is the magnetic Prandtl number. $\eta=y/\sqrt{\cos(\alpha)\nu t+\sin(\alpha)(\nu x/U_w)}$ is the non-dimensional similarity variable. The coordinates for the slot satisfies $y=0$ and $\cos(\alpha)\nu t+\sin(\alpha)(\nu x/U_w)=0$ which shows the slot is moving with the velocity $-U_w\cot(\alpha)$. The corresponding boundary conditions can be written as:

$$\begin{aligned} &f(\eta)=0,\ f'(\eta)=1,\ g(\eta)=0,\ g'(\eta)=1\ \text{ at }\ \eta=0\\ &\qquad f'(\eta)=0,\ g'(\eta)=0\ \text{ as }\ \eta\to\infty \end{aligned} \tag{8}$$

Fang, Zhang and Yao (2010) examined in detail the effects of moving slot parameter α on the velocity and skin friction for unsteady flow of the viscous fluid. In next section we will focus our discussion on the effects of magnetic field parameter Sc on the velocity and skin friction and the effects of α, Sc and Pr on magnetic field and magnetic field flux at the surface.

3. Results and Discussion

As mentioned earlier that in this special case of unsteady flow the slot is moving with constant speed $-U_w \cot(\alpha)$. For $\alpha = \frac{\pi}{2}$, the surface velocity will be zero as in Sakiadis flow (Sakiadis, 1961). As $\alpha \to 0$, the speed of slot approaches infinite value in the direction opposite to the stretching surface, which correspond to Rayleigh starting plate problem. In this case the exact solutions for the momentum and magnetic field boundary layer can be written as:

$$f'(\eta) = 1 - erf\left(\frac{1}{2}\eta\right), \tag{9}$$

$$g'(\eta) = 1 - erf\left(\frac{\sqrt{Pm}}{2}\eta\right). \tag{10}$$

For $0 < \alpha < \frac{\pi}{2}$, the slot will be moving with the constant speed $U_w \cot(\alpha)$ in the opposite direction of stretching surface and the situation is termed as leading edge accretion. For $\alpha \in (\alpha_L,\ 0) \cup \left(\frac{\pi}{2},\ \alpha_U\right)$, the direction of slot motion is same as stretching sheet and the situation is termed as leading edge ablation, where $\alpha_L < \alpha < \alpha_U$ is the numerical domain for the skin friction. Since the exact analytical solution of (6)-(8) is not available for general α, we will adopt the numerical method for the solution.

In [16], Fang *et al.* mentioned that the lower limit for the existence of the solution is at $\alpha_L = -53.55^o$ and the upper limit is about $\alpha_U = 92^o$. This range of α can also be observed from Fig. 2 for Sc = 0.0. It is observed that the magnetic field strongly affects the interval of existence of dual solution specially the lower limit α_L. As the value of Sc increases the interval of existence squeezes and a clear increase α_L is noticed for example, for Sc = 1.0, $\alpha_L \approx -32^o$. For Sc = 0 (Fang, Zhang and Yao, 2010), the peak value of skin friction is noticed at about $\alpha = 32^o$. This peak value moves toward left as the value of Sc increases and for Sc = 1.0, this maxima is observed at $\alpha = -20^o$. In the absence of magnetic field dual solutions exist for α in the interval $\left(\alpha_L,\ -\frac{\pi}{4}\right)$. Interestingly the range of dual solution vanishes with an increase in Sc and no dual solutions are observed even for Sc = 0.1. It is also observed that the skin friction is independent of Sc for $\alpha = 0^o$ since all the curves have common point at $\alpha = 0^o$. Further for $\alpha > 0$, the skin friction is decreasing function of magnetic parameter Sc and for $\alpha < 0$ skin friction increases as the parameter Sc increases. In Figs. 3 and 4, the velocity profile is plotted in the situation of leading edge accretion and ablation for different magnetic field parameter Sc. In the case of leading edge ablation the velocity of fluid decreases as the magnetic field parameter increases. An opposite behavior is observed in the case of leading edge accretion.

In Fig. 5 the dual solutions of velocity (a) and shear stress (b) are plotted. The solution with higher surface shear stress, $-f''(0)$, is termed as upper solution curve and the solution with lower surface shear stress, $-f''(0)$,

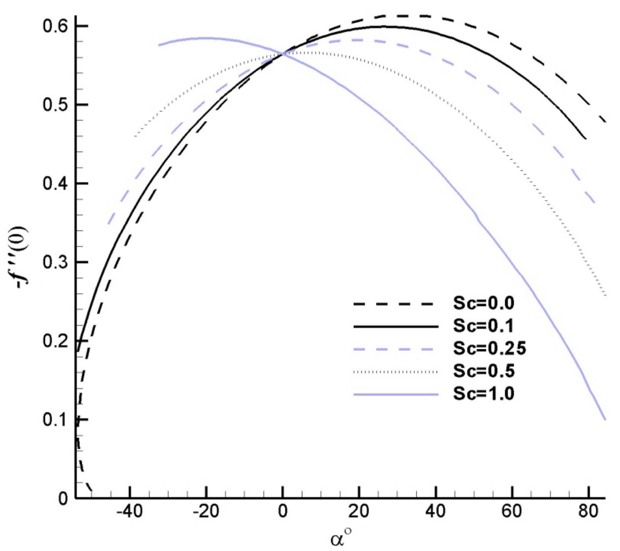


Fig. 2. (Color online) Skin friction verses slot parameter α for different Sc.

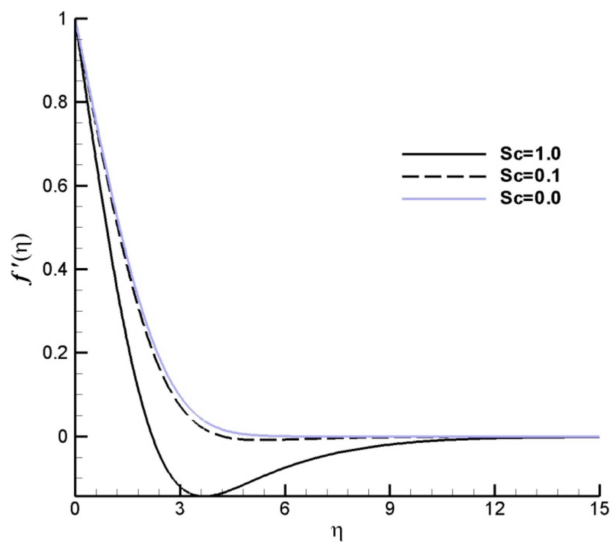


Fig. 3. (Color online) Velocity profile for different Sc with α = −30.

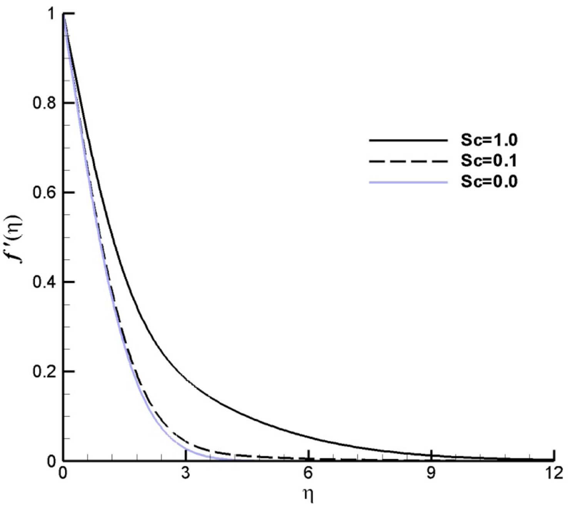


Fig. 4. (Color online) Velocity profile for different Sc with $\alpha = 30$.

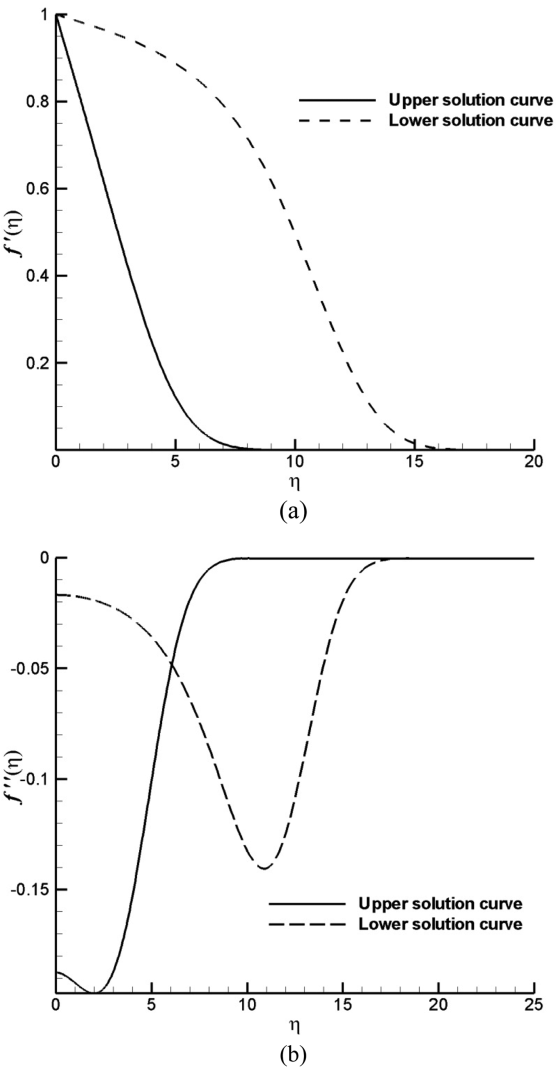


Fig. 5. Different solution curves of velocity and shear stress for $\alpha = -51^\circ$ and $Sc = 0.001$.

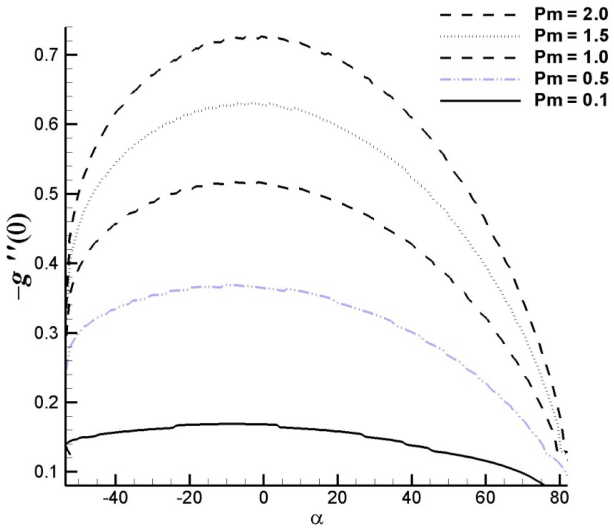
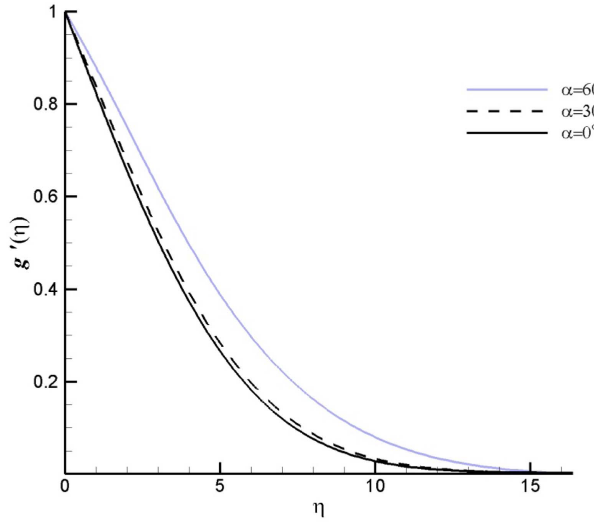


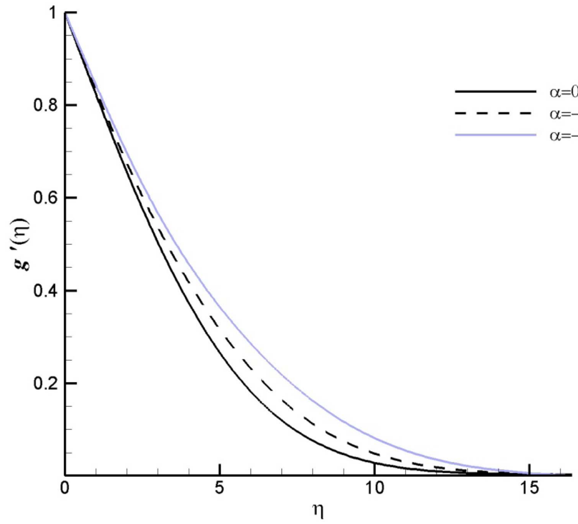
Fig. 6. (Color online) Magnetic flux at surface verses moving slot parameter for different magnetic Prandtl number.

is termed as lower solution curve. It is observed that the boundary layer thickness of lower solution curve is much greater then upper solution curve. In contrast to [19] the shear stress of the upper solution curve is no monotonic. In the neighborhood of the surface the shear stress $-f''(\eta)$ is maximum for the upper solution and minimum for lower solution.

In Fig. 6 the magnetic flux at the surface verses moving slot parameter is plotted for different magnetic Prandtl number Pm. It is observed that magnetic flux at the surface increases with an increase in Pm. Dual solutions exist on the certain range of α for small Pm. The duality of the solution disappears as the value of Pm increases. The magnetic flux at the surface is maximum at $\alpha = 0$, i.e. for Rayleigh starting plate problem. In Fig. 7 magnetic field profiles are plotted for leading edge accretion and ablation cases. It is observed that the thickness of magnetic boundary layer is minimum for $\alpha = 0$. The thickness of magnetic boundary layer increases as $|\alpha|$ increases. In Fig. 8 the upper and lower solution curves of (a) magnetic field and (b) magnetic flux are plotted for different values of Pm. It is observed that the magnetic boundary layer thickness decreases with an increase in Pm due to influence of viscous force over the magnetic force. It is also observed that magnetic flux has maxima at the surface and away from the surface the magnetic flux is zero. The solution with higher magnetic flux at the surface $-g''(0)$ is term as upper solution curve. It is observed that the magnetic boundary layer thickness of lower solution curve is much greater as compare to upper solution curve. Further the lower solution curves are not monotonic and have minima for each Pm in contrast with upper solution curves.

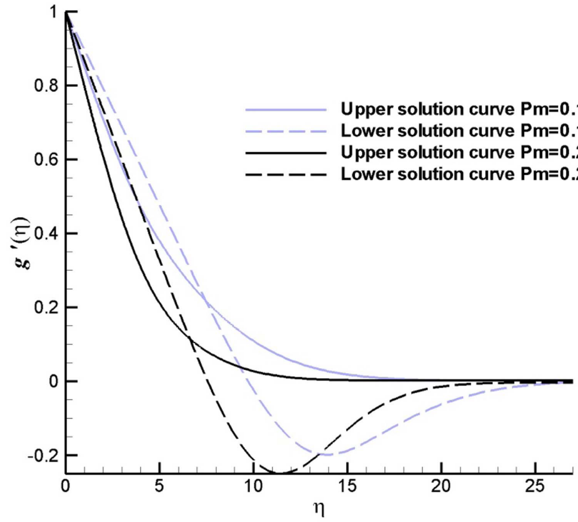


(a)

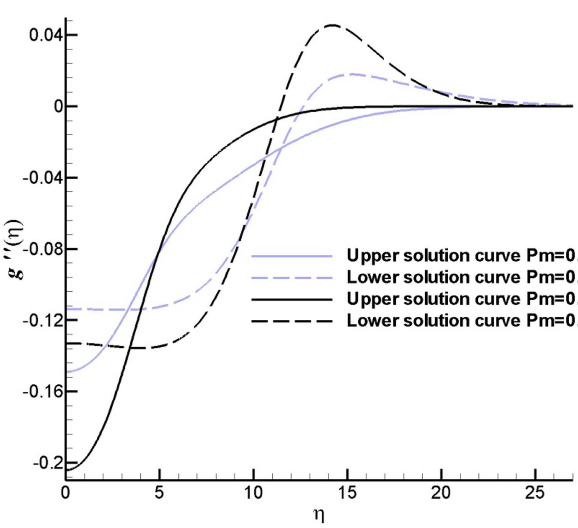


(b)

Fig. 7. (Color online) Magnetic field profile for (a) leading edge accretion and (b) ablation with Pm = 0.1 and Sc = 0.01.



(a)



(b)

Fig. 8. (Color online) Upper and lower solution curves (a) magnetic field profile and (b) magnetic flux for different magnetic Prandrl number with fixed Sc = 0.01 and α = −51°.

4. Conclusion

In this article the unsteady boundary layer flow over a magnetized moving surface emerging from a moving slot with velocity $-U_w \cot(\alpha)$ is studied in detail. This boundary layer problem behaves like stretching and shrinking sheet problem depending on the direction of motion of slot. Exact solution is given for moving slot parameter $\alpha = 0$. The effect of magnetic field due to magnetized surface on the velocity and skin friction is studied and it is concluded that:

1. The magnetic field strongly affects the interval of existence of dual solution.
2. Skin friction is increasing function of magnetic parameter Sc for $\alpha < 0$, decreasing function for $\alpha > 0$ and a constant function for $\alpha = 0$.
3. In the case of leading edge ablation the velocity of fluid decreases as the magnetic field parameter increases. An opposite behavior is observed in the case of leading edge accretion.

The effect of moving slot parameter α and magnetic Prandtl number on magnetic field and magnetic flux at the surface are also examined with the following conclusions:

1. Magnetic flux at the surface is an increasing function of Magnetic Prandtl number.
2. Dual solutions exist on the certain range of α for small Pm. The duality of the solution disappears as the value of Pm increases.

3. The magnetic flux at the surface is maximum at $\alpha = 0$.
4. The thickness of magnetic boundary layer is minimum for $\alpha = 0$.
5. The magnetic boundary layer thickness of lower solution curve is much greater as compare to upper solution curve.

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