ISSN (Print) 1226-1750
ISSN (Online) 2233-6656
Journal of Magnetics 24(3), 408-412 (2019) https://doi.org/10.4283/JMAG.2019.24.3.408

Most Probable Failure Point Update Method for Accurate First-Order Reliability-Based Electromagnetic Designs

Byungsu Kang[1], Jaegyeong Mun[1], Jongsu Lim[1], K. K. Choi[2], and Dong-Hun Kim[1]*

[1]*Dept. of Electrical Eng., Kyungpook National Univ., Daegu 41566, Republic of Korea*
[2]*Dept. of Mechanical and Industrial Eng., Univ. of Iowa, Iowa City, IA 52242-1527, USA*

(Received 22 April 2019, Received in final form 26 August 2019, Accepted 29 August 2019)

A most probable failure point update method is proposed to obtain an accurate reliability-based design of electromagnetic devices or systems in the presence of uncertainties. The first-order reliability method has been recently adopted to solve electromagnetic design problems. However, its result could be very erroneous especially for nonlinear or multi-dimensional performance functions. To overcome the drawback, a three-step computational procedure is additionally executed to ensure prescribed design feasibility at an optimum obtained from the conventional first-order reliability method: failure rate calculation, reliability index update, and most probable point update. A mathematical example and a blushless DC motor design problem are provided to demonstrate numerical accuracy of the proposed method by comparison with the conventional method.

Keywords : Electromagnetics, optimization, reliability theory, robustness

1. Introduction

In recent years, one of probabilistic design methods, called reliability-based design optimization (RBDO), has drawn engineer's attention in our community because it can systematically incorporate uncertainties into an early design stage. The RBDO formulation involves an objective function as deterministic optimization, and also contains probabilistic constraints for taking into account the probability of the satisfaction/failure of constraint functions. Thus, accurate reliability assessment of performance functions concerned in constraint conditions is an essential step in the RBDO process. Up to date, various attempts to quantitatively predict the probability of failure of a performance function have been made in electromagnetic (EM) field analysis and design: first-order reliability method (FORM), moment method, Monte Carlo simulation (MCS) and so on [1-9].

Among them, FORM has been popularly used to evaluate the probability failure rate of an EM performance function because of its simple and efficient implementation [2-4]. In FORM, design random variables are first transformed into the independent and standard normal probability distributions. Then, the performance function is approximated by the first-order Taylor series, and its failure rate is computed by means of a most probable failure point (MPP). According to the MPP search algorithm, there are two different approaches: reliability index approach (RIA) and performance measure approach (PMA). It has been revealed that the FORM-based methods could be very erroneous if the performance function are highly nonlinear [5-7]. That is because FORM approximates the performance function using a liner function, and so it cannot reflect the complexity of nonlinear or high dimensional functions.

As an effort to alleviate the difficulty in the FORM-based methods, the authors proposed a hybrid reliability analysis method for the failure rate calculation of nonlinear or multi-dimensional EM performance functions [8]. Therein, the univariate dimension reduction method (DRM) is incorporated with RIA in order to enhance numerical accuracy of RIA. Through test problems, it has been shown that the method can estimate the probability of failure of a performance function more accurately than RIA and more efficiently than MCS.

In this paper, a wealth of information not used in [8] is exploited to improve numerical accuracy of a solution of the conventional FROM-based RBDO. For the purpose of doing this, a so-called MPP-based DRM is newly used for enhanced RBDO design as well as accurate reliability

*Corresponding author: Tel: +82-53-950-5603
Fax: +82-53-950-5603, e-mail: dh29kim@ee.knu.ac.kr

analysis. A three-step computational procedure is additionally carried out to guarantee prescribed design feasibility at an optimum obtained from the PMA-based RBDO method: failure rate calculation, reliability index update, and most probable point update. Through testing a mathematical example and a brushless DC motor design problem, numerical accuracy of the proposed method is examined by comparison with the conventional PMA-based RBDO method.

2. Three-Step Procedure for Improved RBDO Designs

In general, a probabilistic constraint of the performance function g is expressed by

$$P_f[g(\mathbf{x})>0]=\int_{g(\mathbf{x})>0}\cdots\int f_{\mathbf{x}}(\mathbf{x})\,dx_1\cdots dx_n \le P_f^{tar}=\Phi(-\beta_t). \quad (1)$$

In (1), $P_f(\cdot)$ is the probability of failure for the infeasible condition ($g > 0$), $\mathbf{x}$ is a design random vector in X-space, and $f_{\mathbf{x}}(\mathbf{x})$ is the joint probability density function of all random variables in n dimensional space. The symbol P_f^{tar} is the target probability of failure of g, $\Phi(\cdot)$ is the standard normal cumulative distribution function, and β_t is the target reliability index. For efficient and accurate failure rate computation of (1), the PMA-based reliability analysis is first conducted to find out a MPP in a standard normal space (U-space), and then the univariate DRM is executed at the MPP. In addition, the information generated from such the MPP-based DRM can be used for a more improved next iterative design at the PMA-based RBDO optimum.

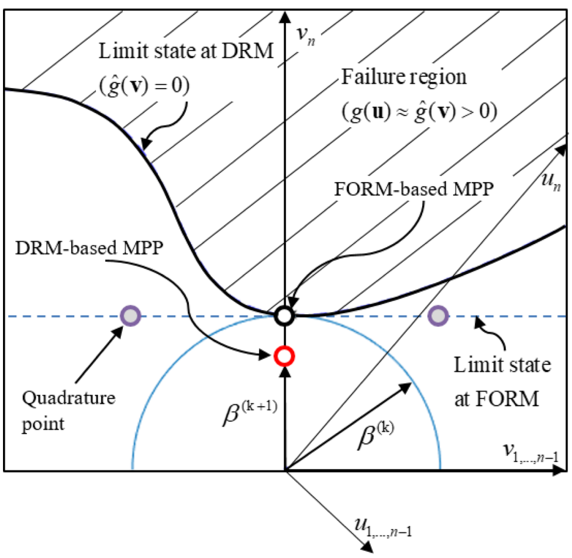


Fig. 1. (Color online) Illustration of DRM-based MPP for a concave performance function.

2.1. Failure Rate Calculation

To handle the multiple integration in (1), PMA adopts the first-order Taylor series to approximate g [9]. The random variable vector $\mathbf{x}$ is transformed to the standard normal random variable vector $\mathbf{u}$. That is, $g(\mathbf{x})$ in X-space is mapped onto $g(T(\mathbf{x})) \equiv g(\mathbf{u})$ in U-space. In PMA, the probabilistic constraint condition of (1) can be dissolved into the inequality equation of (2).

$$P_f(g(\mathbf{x})>0)-\Phi(-\beta_t)\le 0 \Rightarrow g(\mathbf{x}^*)\le 0 \quad (2)$$

where $\mathbf{x}^*$ denotes an inverse MPP in X-space corresponding to the inverse transformation of $\mathbf{u}^*$ in U-space. To seek a MPP $\mathbf{u}^*$, the following optimization problem must be solved.

$$\begin{aligned} &\text{maximize} && g(\mathbf{u}) \\ &\text{subject to} && \|\mathbf{u}\|=\beta_t. \end{aligned} \quad (3)$$

As seen in (3), PMA does not calculate the probability failure rate directly. Instead, it judges whether or not a current design point satisfies probabilistic constraint for a prescribed target failure probability.

To accurately evaluate the probability of failure at MPP $\mathbf{u}^*$, the univariate DRM is additionally conducted. The performance function g is expressed by the sum of one-dimensional ones. To relieve such the approximation error, a rotated standard normal V-space is newly introduced as seen in Fig. 1, where $\mathbf{u}^*$ is defined by $\mathbf{v}^* = [0, \cdots, 0, \beta_t]^T$. The n-dimensional performance function $g(\mathbf{u})$ in U-space is additively decomposed into one-dimension ones at the MPP $\mathbf{v}^*$ in V-space as [7, 8]

$$g(\mathbf{u})\approx\hat{g}(\mathbf{v})\equiv\sum_{i=1}^{n}g_i(v_i)-(n-1)g(\mathbf{v}^*)=g_n(v_n)+\sum_{i=1}^{n-1}g_i(v_i) \quad (4)$$

where $\hat{g}$ is the univariate approximation function, $g_i(v_i) = g(0, \cdots, 0, v_i, 0, \cdots, \beta_t)$ is a function of only v_i. Due to the rotational transformation of the coordinates, the nth univariate component $g_n(v_n)$ can be linearly approximated along v_n-axis as

$$\begin{aligned} g_n(v_n) &\cong g_n(v_n^*)+\partial g(\mathbf{v})/\partial v_n\big|_{\mathbf{v}=\mathbf{v}^*}(v_n-v_n^*) \\ &=\partial g(\mathbf{v})/\partial v_n\big|_{\mathbf{v}=\mathbf{v}^*}(v_n-\beta_t)=b_1(v_n-\beta_t) \end{aligned} \quad (5)$$

where $g_n(v_n^*)$ is zero because of $v_n^* = \beta_t$, and b_1 denotes the partial derivative of $\partial g(\mathbf{v})/\partial v_n\big|_{\mathbf{v}=\mathbf{v}^*}$.

Using the linear assumption of (4) and (5), the probability of failure in (2) can be rewritten as

$$\begin{aligned} P_f &= P[g(\mathbf{x})>0]\cong P[\hat{g}(\mathbf{v})>0]=P[v_n<-\beta_t+\frac{1}{b_1}\sum_{i=1}^{n-1}g_i(v_i)] \\ &=E[\Phi(-\beta_t+\frac{1}{b_1}\sum_{i=1}^{n-1}g_i(v_i))] \end{aligned} \quad (6)$$

where E denotes the expectation operator. Applying the moment-based integration rule (MBIR) to (6), a DRM-based failure rate P_f^{DRM} of g can be generalized as follows [8].

$$P_f^{DRM} = \prod_{i=1}^{n} \sum_{j=1}^{m} w_j \Phi(-\beta_t + g_i(v_i^j)/b_1) / \Phi(-\beta_t)^{n-2} \qquad (7)$$

where m is the number of weights and quadrature points, and the symbols, w_i^j and v_i^j, mean the jth weight factor and quadrature point for the ith random variable v_i, respectively. According to the MBIR theory, m quadrature points and weights yield a degree of precision of $2m-1$.

2.2. Reliability Index Update

The FORM-based reliability assessment like (3) inherently includes a significant error due to the linear approximation of the original limit state function ($g(\mathbf{u}) = 0$), which is highly nonlinear as seen in Fig. 1. Especially for a concave performance function, PMA overestimate the failure rate of g because the approximated failure region above the dash line becomes much larger than the original one ($g(\mathbf{u}) > 0$) marked with oblique lines. It means that the target reliability index β_t must be corrected to reflect such the numerical computation error occurred in PMA.

For doing this, the reliability index β_{DRM} is derived from the accurate failure rate computation of (7) like (8)

$$\beta_{DRM} = -\Phi(P_f^{DRM}). \qquad (8)$$

It is natural that this β_{DRM} is not the same as the target reliability index β_t. To define a new updated reliability index $\beta^{(k+1)}$, a recursive formula is obtained using the difference between the two reliability indices as follows.

$$\beta^{(k+1)} \cong \beta^{(k)} - (\beta_{DRM} - \beta_t) \qquad (9)$$

where $\beta^{(k)}$ is the reliability index at the current step with $\beta^{(0)} = \beta_t$ at the initial step. The updating process is repeated until the failure rate P_f^{DRM} obtained using the MPP-based DRM is the same as the given target failure rate P_f^{tar}.

2.3. MPP Update

According to (9), a new MPP has to be sought out in U-space by solving (3), where the reliability index β_t is replaced with the updated reliability index $\beta^{(k+1)}$. However, since such the search process could be computationally expensive, the updated MPP is approximated without carrying out a new MPP search as

$$\mathbf{u}_{k+1}^* \cong (\beta^{(k+1)}/\beta^{(k)})\mathbf{u}_k^*. \qquad (10)$$

That is, it is assumed that the updated MPP u_{k+1}^* called the DRM-based MPP is located along the same radial direction as the current MPP u_k^* as illustrated in Fig. 1. If the current design is a RBDO optimum, the probability of failure by DRM will converge to the target failure rate along with only a few three-step procedures.

3. Case Studies

To examine the accuracy and usefulness of the proposed MPP-based DRM, two RBDO problems are considered. Therein, somewhat unsatisfactory RBDO optima are enhanced by means of the three-step computational procedure, where three quadrature points are adopted. The first is a two-dimensional mathematical design problem, and the second is a practical EM design problem, where a reliability-based design of a BLDC motor with six design random variables is attempted.

3.1. Mathematical Model

A RBDO formulation of the mathematical problem is given by

$$\begin{aligned} &\text{min.}\ \ f(\mathbf{d}) = -(d_1 + d_2 - 10)^2/30 - (d_1 - d_2 + 10)^2/120 \\ &\text{subject to}\quad P_f(g_i(\mathbf{x}) > 0) \le 0.135\% \qquad i = 1,2,3 \\ &\qquad g_1(\mathbf{x}) = 1 - x_1^2 x_2/20 \\ &\qquad g_2(\mathbf{x}) = -1 + (y-6)^2 + (y-6)^3 - 0.6(y-6)^4 + z \\ &\qquad g_3(\mathbf{x}) = 1 - 80/(x_1^2 + 8x_2 + 5) \\ &\qquad y = 0.9063x_1 + 0.4226x_2, \quad z = 0.4226x_1 + 0.9063x_2 \end{aligned} \qquad (11)$$

where $\mathbf{d}$ is a design variable vector defined by $\mathbf{d} = \mu(\mathbf{x})$, where μ denotes the mean of $\mathbf{x}$. The random variables are assumed to comply with normal probability distributions with a standard deviation (SD) value of 0.3. The target probability of failure of three constraints is set to be 0.135 %.

In order to reduce a computational cost required for

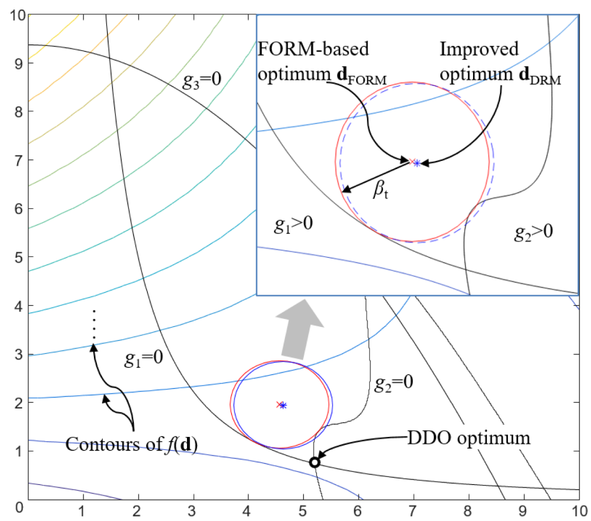


Fig. 2. (Color online) Three different optima and limit-state constraint functions.

Table 1. Performances Indicators between Four Different Design Points.

Design variables	Initial	DDO	RBDO	
			Conventional	Proposed
x_1	5.0	4.850	4.563	4.626
x_2	5.0	0.850	1.962	1.943
f	–0.833	–3.708	–1.723	–1.733
$P_f(g_1)$ %	0.00	51.80	0.156	0.137
$P_f(g_2)$ %	0.00	43.04	0.070	0.138
$P_f(g_3)$ %	1.31	0	0	0
Iteration/ Function calls	-	4/52	7/2472	2/90

The failure rates were recalculated at four different design points by MCS with 200,000 samples.

RBDO, a deterministic design optimization (DDO) corresponding to (11) was first executed with an initial point (5, 5). Launching at the obtained DDO point, the RBDO problem of (11) was solved by the conventional PMA-based RBDO method. Then, its optimum was corrected by the proposed method so as to meet the target failure rates of given probabilistic constraints. Three different optima are compared with each other in Fig. 2, where two circles have the same radius of a target reliability index $\beta_t = \Phi^{-1}(0.135)$ at the center of each RBDO optimum. Table 1 presents performance indicators between four different designs. When engaged in randomness of design variables, the failure rates of g_1 and g_2 at the DDO point reach up to 51.8 % and 43.04 %, respectively.

Meanwhile, the failure rates of g_1 and g_2 at the conventional RBDO optimum $\mathbf{d}_{FORM}$ are close to the target value 0.135 %, but the failure rate of g_2 slightly violates the prescribed probability constraint condition due to the high nonlinearity of g_2. It is observed that such the somewhat unsatisfactory result was substantially alleviated by the proposed MPP-based DRM after only two iterative designs. As shown in Fig. 2, it can be identified that the improved RDBO optimum $\mathbf{d}_{DRM}$ slightly moves towards the infeasible region of g_2 in order to compensate for the overestimated failure rate in the PMA process.

3.2. BLDC Motor Design

A 5 kW, 8-pole and 12-slot BLDC motor with immersed permanent magnets for electric vehicles is considered as in Fig. 3. The outer diameter of a stator is 200 mm, lamination stack height 38 mm, outer diameter of a rotor 126 mm, and air-gap length 0.7 mm. In general, a bridge part on the rotor core formed due to the flux barrier (refer to x_1 in Fig. 3) is highly saturated under normal operating conditions. A commercial EM simulator called MagNet VII was hereby used to accurately estimate the cogging torque without an electric loading condition and average torque at a rated speed of 2,300 rpm [10]. To alleviate a computationally heavy burden on the RBDO process, a fourth model of the motor was analyzed with a periodic boundary condition, and a sinusoidal current was fed to the stator coils without a motor drive system.

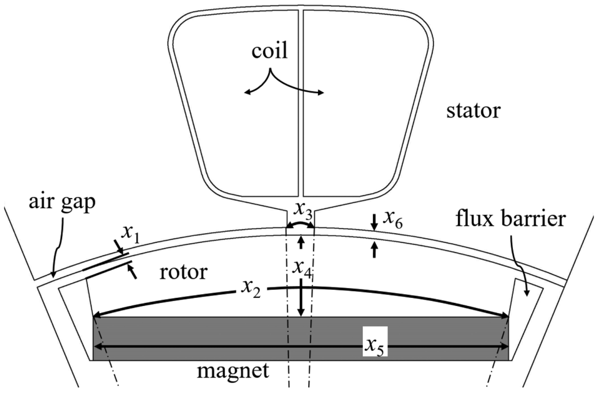
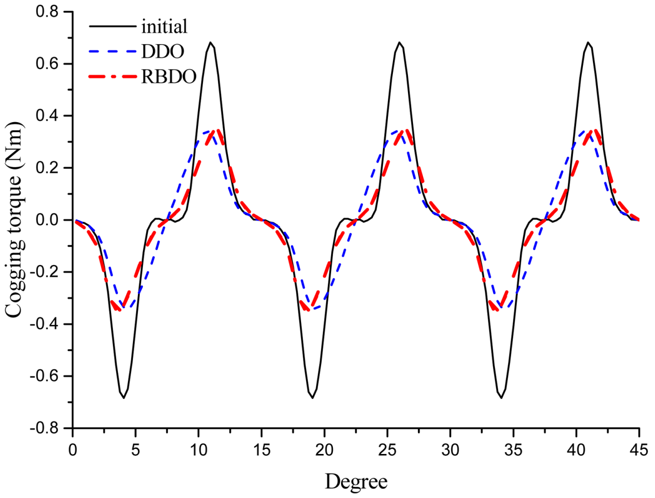


Fig. 3. One-eighth model and six design random variables.

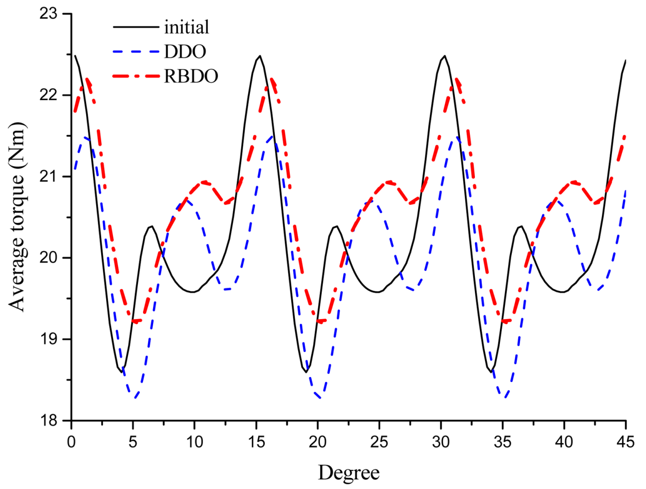
For the purpose of reducing acoustic noise and mechanical vibration of the motor in the present of manufacturing tolerances, the design goal is set to minimize the cogging torque magnitude T_c, and also to satisfy two probabilistic constraint conditions. The two constraints are related to the average torque T_{avg} and torque ripple at the rated speed. Six design random variables are assumed to follow to normal probability distributions with a SD value of 0.1. The wanted failure rates of two constraints is set to be 5 % (i.e. reliability of 95 %) under the given design uncertainty. To manage these somewhat complicated requirements, a RBDO formulation can be written by

Table 2. Performances Indicators between Four Different Designs.

Design variables	Initial	DDO	RBDO	
			Conventional	Proposed
x_1 (mm)	0.80	1.51	1.640	1.639
x_2 (degree)	36.70	37.42	37.920	37.920
x_3 (degree)	2.24	2.06	1.800	1.799
x_4 (mm)	7.00	4.88	5.220	5.222
x_5 (mm)	38.00	36.00	37.760	37.751
x_6 (mm)	0.7	0.67	0.590	0.587
Cogging torque T_c (Nm)	0.684	0.344	0.347	0.347
Average torque T_{avg} (Nm)	20.52	20.01	20.67	20.68
Torque ripple %	23.4	15.7	13.4	13.7
$P_f(g_1)$ %	-	49.09	5.88	5.00
$P_f(g_2)$ %	-	31.57	3.79	3.28
Iteration/FEA calls	-	10/227	17/879	1/65



(a)



(b)

Fig. 4. (Color online) Comparison of torque waveforms between three different designs: (a) Cogging torque, (b) Torque ripple.

$$\begin{aligned} &\text{minimize} \quad f(\mathbf{d}) = T_c(\mathbf{d}) \\ &\text{subject to} \quad P_f(g_i(\mathbf{x})>0) \le 5\%, \quad g_1(\mathbf{x}) = 20 - T_{avg}(\mathbf{x}), \\ &\qquad\qquad g_2(\mathbf{x}) = (T_{\max}(\mathbf{x}) - T_{mim}(\mathbf{x}))/T_{avg}(\mathbf{x}) - 0.177 \end{aligned} \tag{12}$$

where $T_{\max}$ and $T_{\min}$ denote the maximum and minimum torque values, respectively.

Launching at an initial motor design, three optimum designs (DDO, conventional RBDO, and improved RBDO) were obtained one by one according to the same design procedure used in the previous test model. The performance indicators between four different designs of the BLDC motors are presented in Table 2, where failure rate values at three optima were reevaluated by the first step process of the proposed MPP-based DRM. It is observed that the cogging torque and torque ripple amplitudes at the optimum designs are much smaller than those at the initial one as seen in Fig. 4, where torque waveforms were calculated as the rotor rotated from 0 to 45 degrees. When involved with the randomness of design variables, however, the DDO design violates the two given probabilistic constraints by more than 30 %. It implies that the average torque and torque ripple of the DDO design cannot satisfy the confidence level of 95 % under the aforementioned manufacturing tolerance. On the other hand, the conventional PMA-based RBDO design satisfies the second probabilistic constraint, but slightly violates the first one. It is obvious that the proposed RBDO design fulfills both two probabilistic constraints after only one iterative design, which requires 65 finite element analysis (FEA) simulations.

4. Conclusion

This paper proposes the MPP-based DRM for accurate failure rate evaluation of a highly nonlinear EM performance function and accordingly improved RBDO optimum in the present of uncertainties. Results show that the proposed method can successfully compensate for numerical computation errors appearing in the conventional PMA-based RBDO process.

Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2017R1A2B4008046).

References

[1] A. Haldar and S. Mahadevan, Probability, reliability, and statistical methods in engineering design, John Wiley & Sons, New York/Chichester, UK, 2000.
[2] B. Youn and K. Choi, J. Mech. Des. **126**, 1 (2004).
[3] B. Youn, K. Choi, and L. Du, J. Struct. Multidisc. Optim. **29**, 134 (2005).
[4] D. Kim, N. Choi, C. Lee, and D. Kim, IEEE Trans. Magn. **50**, 7016804 (2014).
[5] I. Lee and K. Choi, Struct. Multidisc. Optim. **44**, 299 (2011).
[6] D. Kim, N. Choi, C. Lee, and D. Kim, IEEE Trans. Magn. **51**, 7205104 (2015).
[7] I. Lee, K. Choi, L. Du, and D. Gorsich, J. Comput. & Struct. **86**, 1550 (2007).
[8] B. Kang, C. Kim, H. Cho, K. Choi, and D. Kim, IEEE Trans. Magn. **54**, 7000804 (2018).
[9] B. Kang, D. Kim, K. Choi, and D. Kim, IEEE Trans. Magn. **53**, 7000604 (2017).
[10] MagNet User's Manual, Infolytica Corporation, Quebec, Canada, 2008.